ASTR 400/700: Stellar Astrophysics

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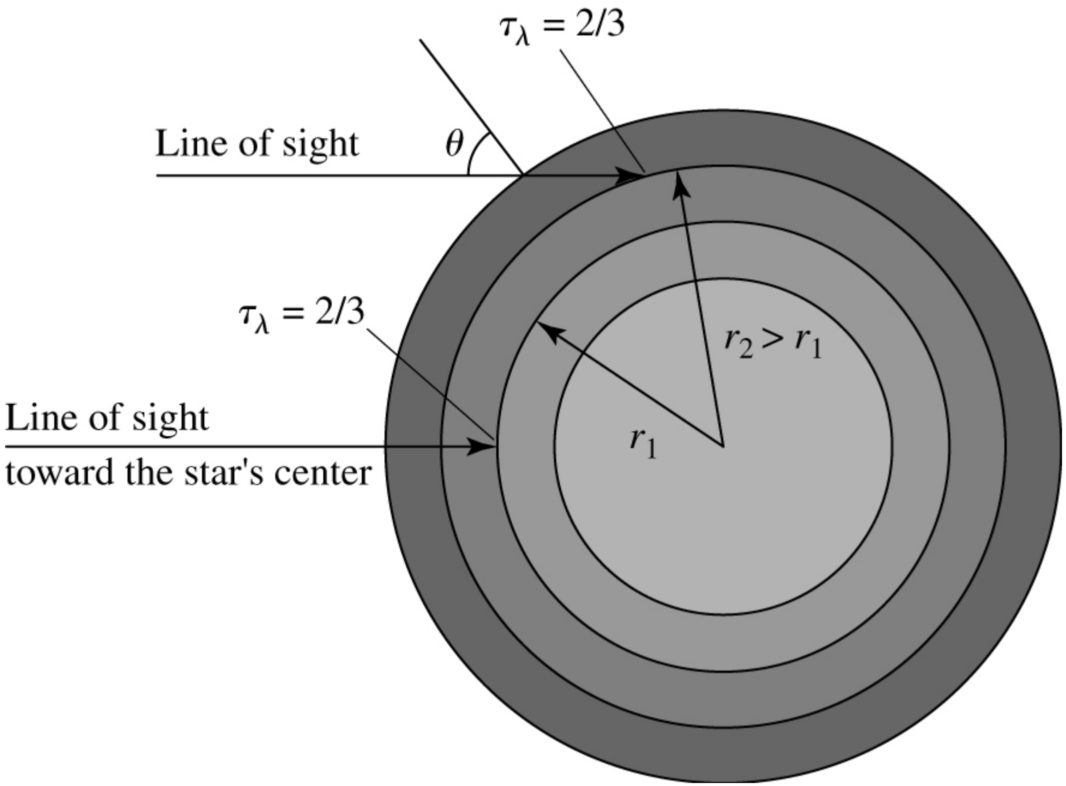
Stellar Atmospheres

Chapter 9.4, 9.5

Limb Darkening

- When looking at the center of the sun one can see "deeper" than when looking at the edge of the sun
    - Deeper is hotter
    - Hotter is brighter…

LIMB DARKENING



The Transfer Equation

Describes the passage of light through a Star's Atmosphere

- The Emission Coefficient j_λ
  - Describes the rate of emission processes

$$dI_\lambda = j_\lambda \rho \, ds,$$

- Source of radiation $S_\lambda = j_\lambda/\kappa_\lambda$
  - Ratio of rates of emission to absorption

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda \, ds + j_\lambda \rho \, ds.$$

$$-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - \frac{j_\lambda}{\kappa_\lambda}.$$

- Equation of Radiative Transport

$$\boxed{-\frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda.}$$

Plane-Parallel Atmosphere

- Far from center of star the radius of curvature is large…consider plane-parallel (flat) slabs…
  - Work in units of optical depth instead of distance…

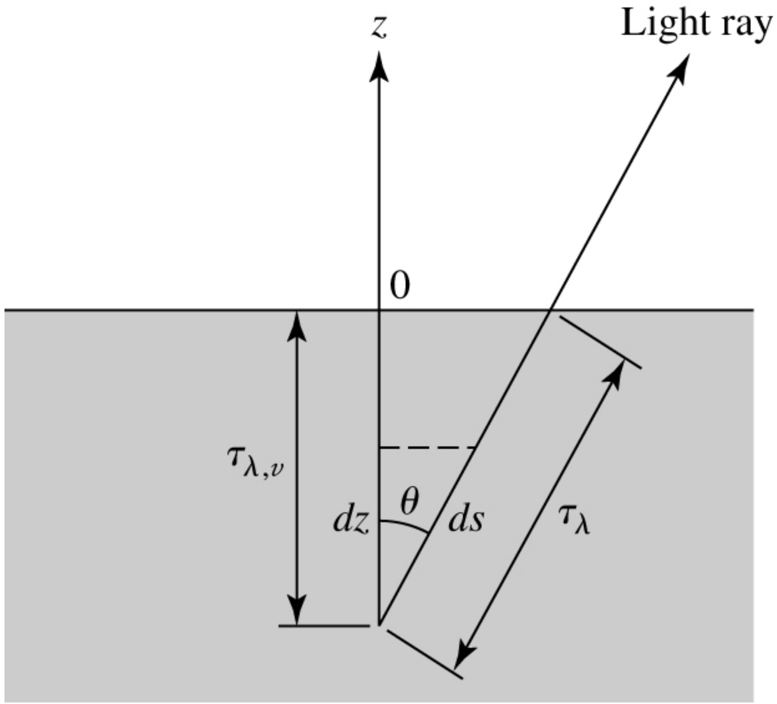
$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda .$$

- Z-axis is vertical z=0 is at top of slab
  - Vertical Optical Depth

$$\tau_{\lambda,v}(z) \equiv \int_z^0 \kappa_\lambda \rho \, dz.$$

Further simplify by using mean opacity--->"Gray Atmosphere". No more dependence on wavelength.

In an equilibrium stellar atmosphere, every process of absorption is balanced by an inverse process of emission, **no net energy is subtracted from or added to the radiation field.**

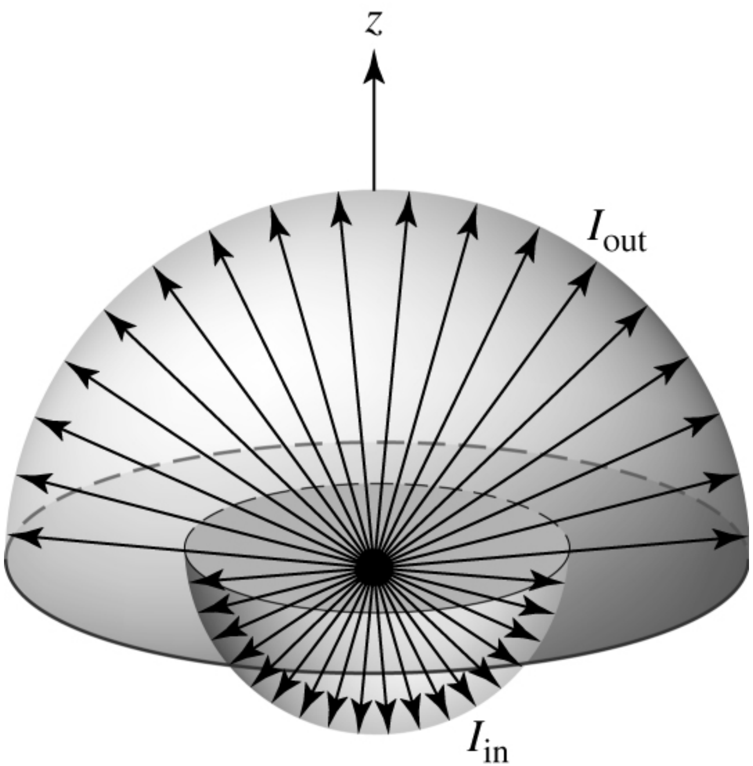


Eddington Approximation

- Approximate I with I_{out} and I_{in}
- Evaluate Temperature as a function of optical depth

$$\langle I \rangle = \frac{3\sigma}{4\pi} T_e^4 \left(\tau_v + \frac{2}{3} \right).$$

$$T^4 = \frac{3}{4} T_e^4 \left(\tau_v + \frac{2}{3} \right).$$



can be generalized to the statement that *when looking at a star, we see down to a vertical optical depth of* $\tau_v \approx 2/3$*, averaged over the disk of the star*. The importance of this for the formation and interpretation of spectral lines was discussed on page 254.

Limb Darkening Revisited

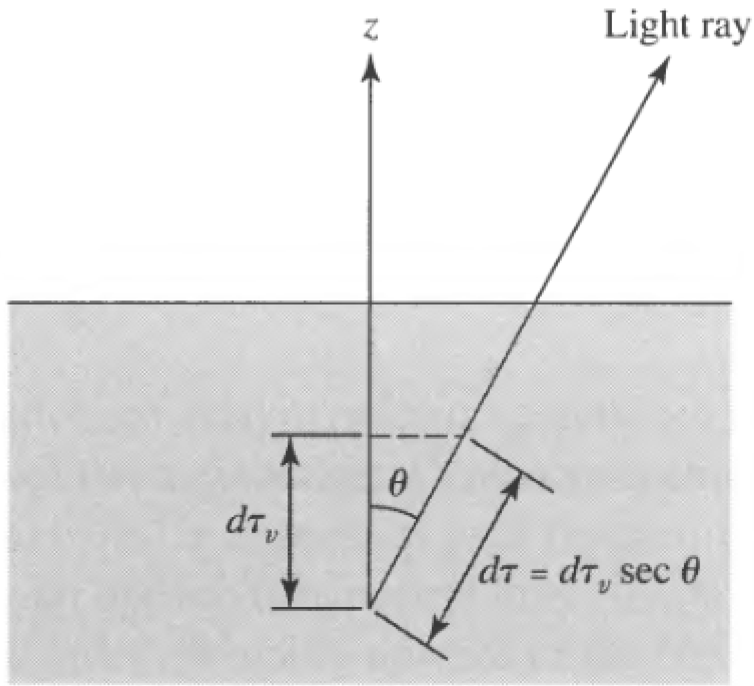
$$\frac{dI_\lambda}{d\tau_\lambda} = I_\lambda - S_\lambda,$$

$$\frac{dI_\lambda}{d\tau_\lambda} e^{-\tau_\lambda} - I_\lambda e^{-\tau_\lambda} = -S_\lambda e^{-\tau_\lambda}$$

$$\frac{d}{d\tau_\lambda}(e^{-\tau_\lambda} I_\lambda) = -S_\lambda e^{-\tau_\lambda}$$

$$d(e^{-\tau_\lambda} I_\lambda) = -S_\lambda e^{-\tau_\lambda}\, d\tau_\lambda.$$

$$I_\lambda(0) = I_{\lambda,0} e^{-\tau_{\lambda,0}} - \int_{\tau_{\lambda,0}}^{0} S_\lambda e^{-\tau_\lambda}\, d\tau_\lambda.$$



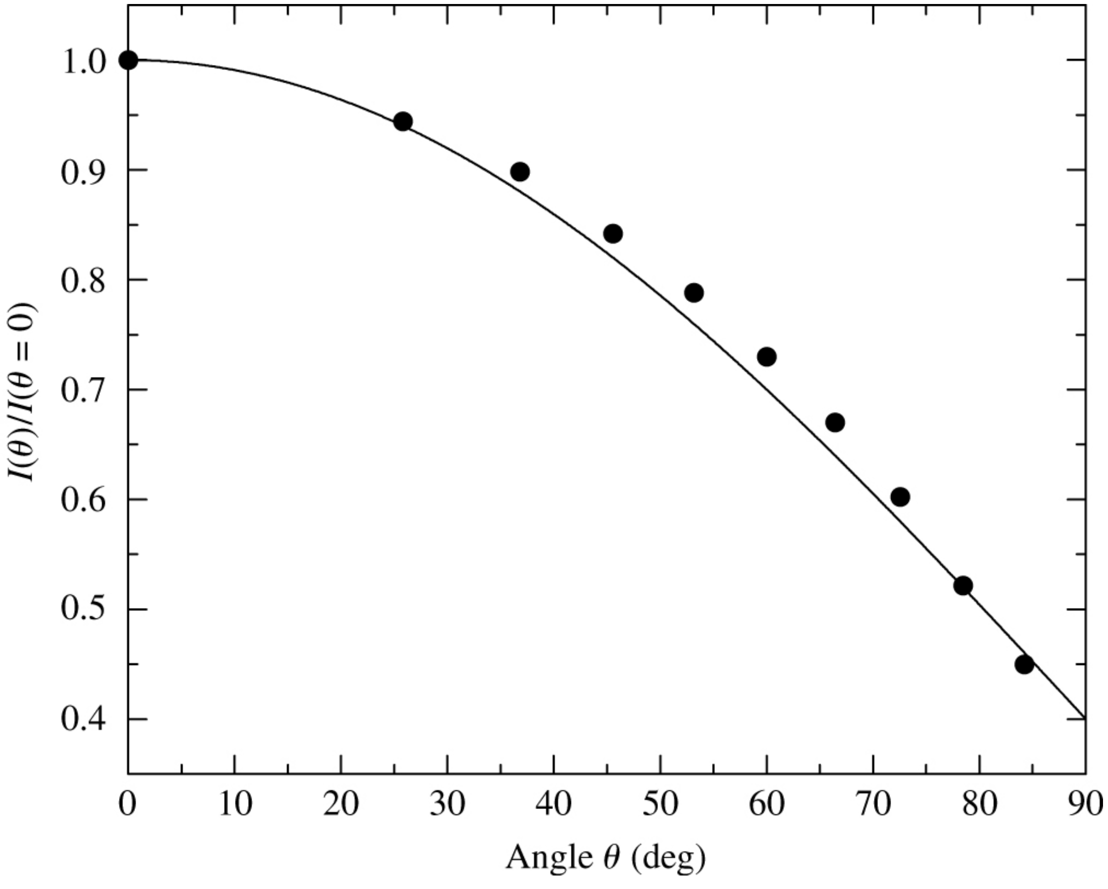
$$I(0) = I_0 e^{-\tau_{v,0}\sec\theta} - \int_{\tau_{v,0}\sec\theta}^{0} S \sec\theta\, e^{-\tau_v \sec\theta}\, d\tau_v.$$

$$I(0) = \int_0^\infty S \sec\theta\, e^{-\tau_v \sec\theta}\, d\tau_v.$$

$$S = a + b\tau_v,$$

$$I_\lambda(0) = a_\lambda + b_\lambda \cos\theta,$$

Eddington Approximation of Solar Limb Darkening



Profiles of Spectral Lines

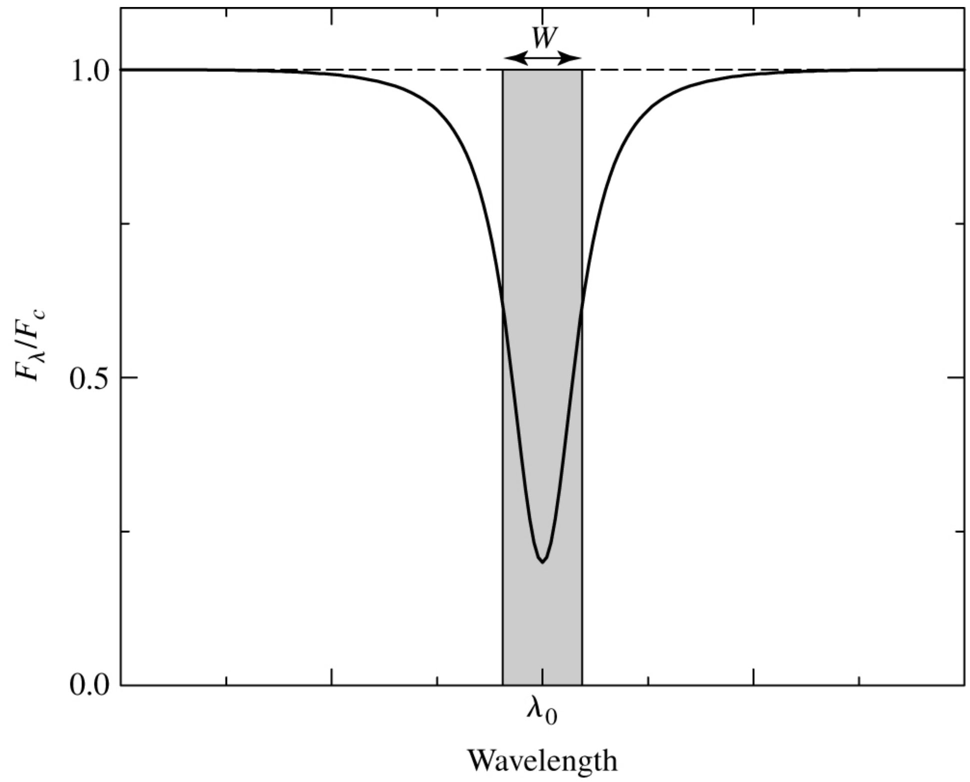
- Spectral Line conveys information about the environment in which it was formed
  - Line width

$$W = \int \frac{F_c - F_\lambda}{F_c}\, d\lambda,$$

  - Line Shape

– Broadening

– Profile



Optically Thin Spectral Line. Thus termed because there is no wavelength at which the radiant flux has been **completely** blocked.

Sources of Broadening

- **Natural Broadening**

– From Uncertainty Principle

– ~2-4 x 10^{-5} nm

- **Doppler Broadening**

– From thermal motion of atoms

– ~4.27 x 10^{-2} nm at 5772K

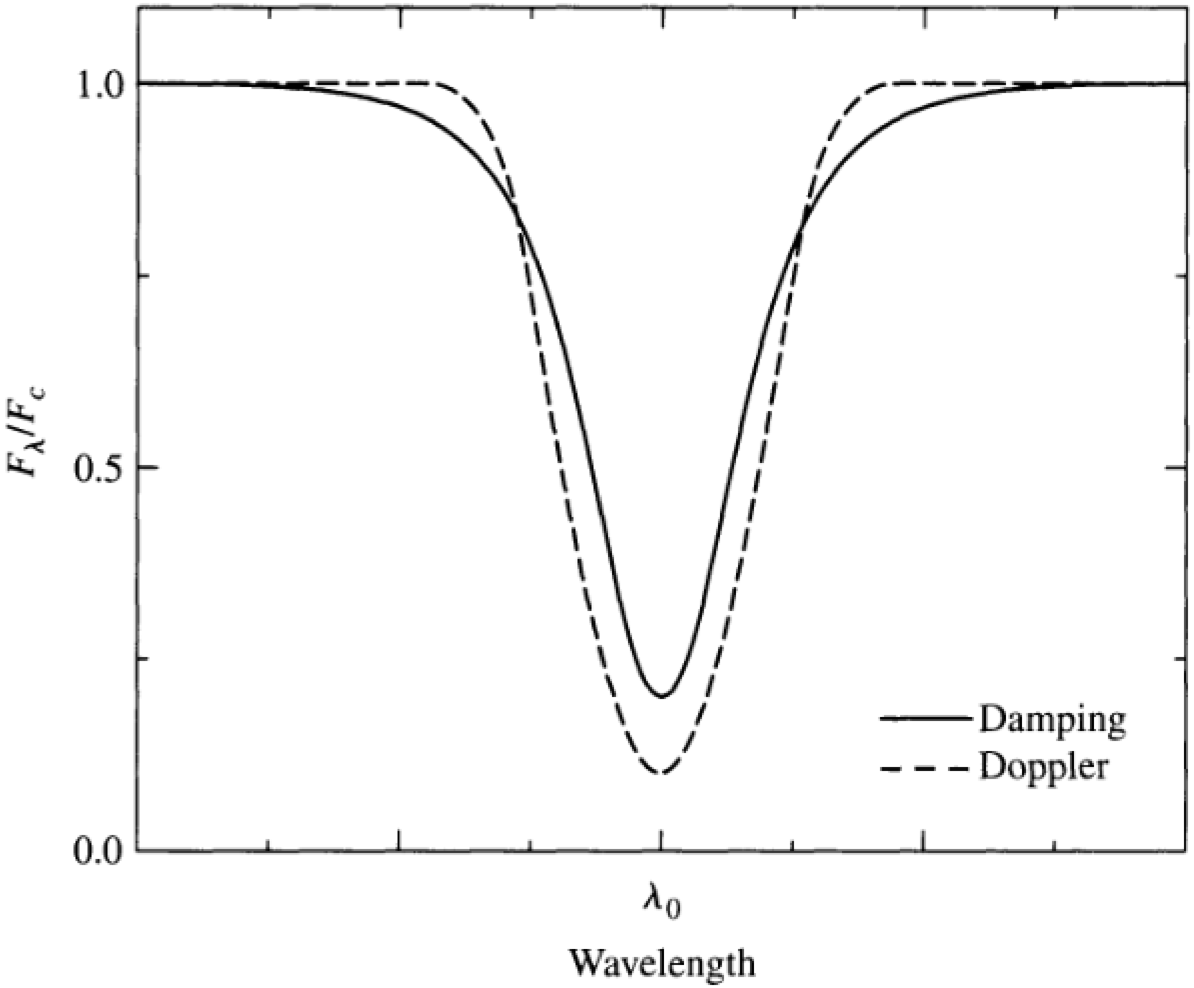
- **Pressure and Collisional Broadening**

– From atomic orbitals being perturbed from collisions

– ~2-4 x 10^{-5} nm

The Voigt Profile

- Combination of all broadening effects



The Voigt Profile

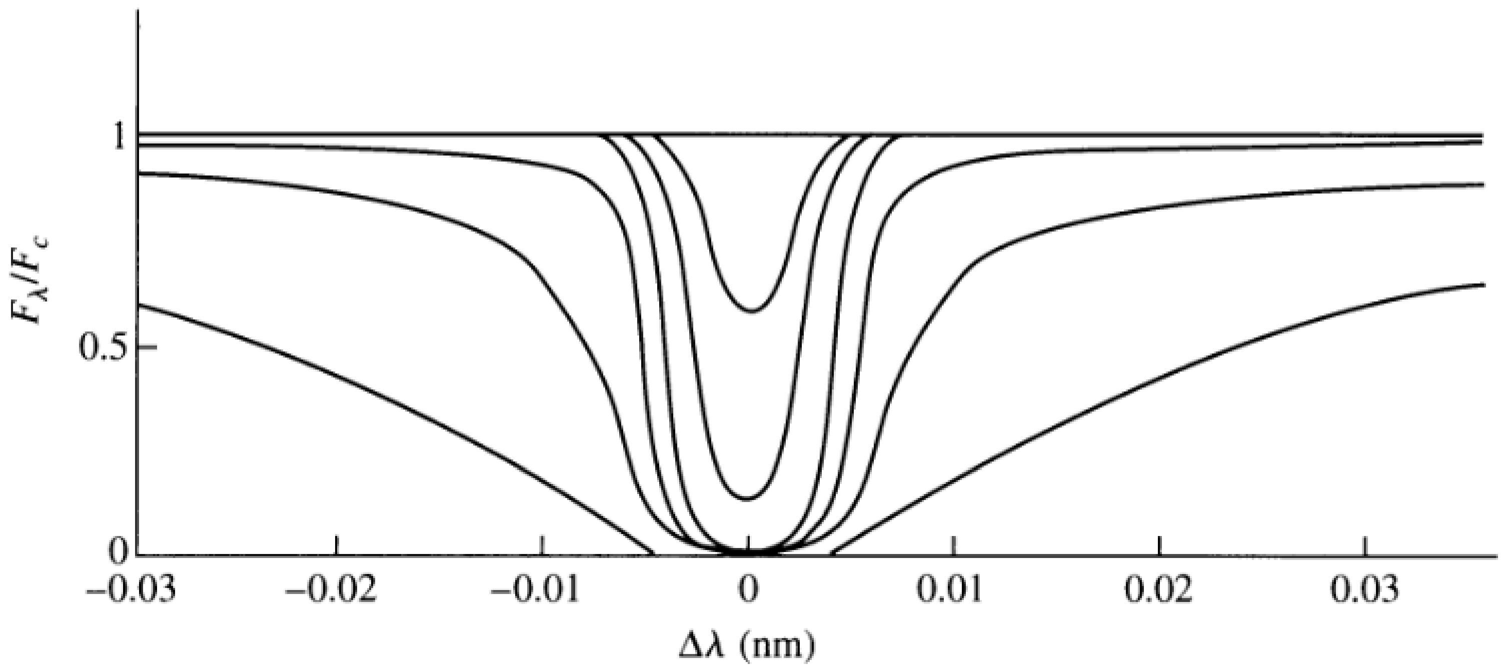


FIGURE 9.20 Voigt profiles of the K line of Ca II. The shallowest line is produced by $N_a = 3.4 \times 10^{15}$ ions m^{-2}, and the ions are ten times more abundant for each successively broader line. (Adapted from Novotny, *Introduction to Stellar Atmospheres and Interiors*, Oxford University Press, New York, 1973.)