

ASTR 400/700: Stellar Astrophysics

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Stellar Interiors

Chapter 10.4, 10.5, 10.6

Stellar Structure

- Variation of temperature
- Variation of density
- Variation of pressure

Test against observations with computer models.

Stellar evolution results from a constant battle with gravity!

Physical Principles

- Hydrostatic Equilibrium
- Virial Theorem
- Energy Generation
- Energy Transport

Models of Stars

The parameters used for studying and modeling stellar interiors include:

r = radial distance from the centre of the star

$M(r)$ = mass interior to r

$T(r)$ = temperature at r

$P(r)$ = pressure at r

$L(r)$ = luminosity at r

$\epsilon(r)$ = energy generation at r

$\kappa(r)$ = opacity at r

$\rho(r)$ = density at r

In modern models mass M is used as the dependent variable rather than radial distance r , but it is more informative to initiate the study of stellar interiors using the geometrical variable r .

Nuclear Energy Sources

Consider the rest masses of the fundamental nuclear particles:

Proton: 1.672623×10^{-24} gm

Neutron: 1.674929×10^{-24} gm

Electron: 9.109390×10^{-28} gm

Atomic mass unit, $1 u = 1.660540 \times 10^{-24}$ gm = 931.49432 MeV,
for $E = mc^2$.

The original nucleon symbolism was: ${}^A_Z X$

where **A** = mass number = number of nucleons

Z = number of protons (usually omitted)

X = chemical symbol of the element as specified by **Z**.

i.e., ${}^1_1\text{H}$ is redundant, since ${}^1\text{H}$ indicates the same thing.

Typical masses:

$${}^1\text{H} = 1.007825 \text{ } u = 938.78326 \text{ MeV}$$

$${}^2\text{H} = 2.014102 \text{ } u = 1876.12457 \text{ MeV}$$

$${}^4\text{He} = 4.002603 \text{ } u = 3728.40196 \text{ MeV}$$

$${}^5\text{Li} = 5.0125 \text{ } u = 4669.115279 \text{ MeV}$$

$${}^8\text{Be} = 8.005305 \text{ } u = 7456.89614 \text{ MeV}$$

The major reaction in astronomy converts 4 hydrogen nuclei (protons) into a helium nucleus (${}^4\text{He}$).

$$\text{But } 4 \text{ } {}^1\text{H} = 1.007825 \text{ } u \times 4 = 4.031280 \text{ } u$$

$$\text{and } 1 \text{ } {}^4\text{He} = 4.002603 \text{ } u = 4.002603 \text{ } u$$

$$\text{Difference} = 0.028677 \text{ } u = 0.0071 \text{ of } 4 \text{ } {}^1\text{H}$$

$$\begin{aligned} \text{The energy released} &= mc^2 = 0.028677 \text{ } u \times 1.660540 \times 10^{-24} \text{ gm } c^2 \\ &= 26.71 \text{ MeV.} \end{aligned}$$

The lifetime of a star depends upon how much hydrogen content is converted to energy via nuclear reactions.

For the Sun we can estimate:

$$E_{\text{nuclear}} = 0.10 \times 0.0071 \times M_{\text{Sun}} c^2 = 1.3 \times 10^{51} \text{ ergs}$$

At $L = 3.851 \times 10^{33}$ ergs/s,

$$t_{\text{nuclear}}(\text{Sun}) = \frac{E_{\text{nuclear}}}{L_{\text{Sun}}} = \frac{1.3 \times 10^{51} \text{ ergs}}{3.851 \times 10^{33} \text{ ergs/s}} = \frac{3.38 \times 10^{17} \text{ s}}{3.1557 \times 10^7 \text{ s/yr}} \cong 10^{10} \text{ yr}$$

1 M , $t_{\text{nuclear}} = 10^{10}$ years 2 M , $t_{\text{nuclear}} = 10^9$ years (A-star)

4.6 M , $t_{\text{nuclear}} = 10^8$ years 10 M , $t_{\text{nuclear}} = 10^7$ years (B-star)

21.5 M , $t_{\text{nuclear}} = 10^6$ years (O-star)

0.5 M , $t_{\text{nuclear}} = 10^{11}$ years $> 1/H_0$ (estimated age of the universe)

The lifetime of the Sun and stars via nuclear reactions is consistent with the nuclear ages of meteorites, as well as the oldest rocks on the Earth and the Moon.

The energy released by nuclear reactions per gram of stellar material is given by:

$$\epsilon_{i,x} = \left(\frac{\epsilon_0}{\rho} \right) r_{ix} \quad \text{or}$$

$$\epsilon_{i,x} = \epsilon'_0 X_i X_x \rho^\alpha T^\beta$$

where $\alpha = \alpha' - 1$. The units are ergs/gm/s.

Energy generated through nuclear reactions is responsible for a star's luminosity, through the equation of continuity:

$$\frac{dL}{dr} = 4\pi r^2 \rho \epsilon$$

where $\epsilon = \epsilon_{\text{nuclear}} + \epsilon_{\text{gravity}}$, where the latter term is not always negligible.

Energy Transport and Thermodynamics:

So far we have developed the following equations of stellar structure:

Equation of Continuity:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho$$

Hydrostatic Equilibrium:

$$\frac{dP}{dr} = \frac{-G M(r) \rho}{r^2}$$

Energy Generation:

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

But what about the temperature gradient, dT/dr ?

Radiative Transport:

Energy can be transported through a star by radiation, convection, or conduction. Conduction is unimportant in most gaseous stellar interiors, but the other two processes are important. In stellar atmospheres radiative energy transport is described by:

$$\frac{dP_{\text{rad}}}{dr} = -\frac{\bar{\kappa} \rho}{c} F_{\text{rad}}$$

But $P_{\text{rad}} = \frac{1}{3}aT^4$, so:

$$\frac{dP_{\text{rad}}}{dr} = \frac{4aT^3}{3} \frac{dT}{dr} = -\frac{\bar{\kappa} \rho}{c} F_{\text{rad}}$$

Thus:

$$\frac{dT}{dr} = -\frac{3 \bar{\kappa} \rho}{4ac} F_{\text{rad}} T^3$$

But $F_{\text{rad}} = L_r/4\pi r^2$, giving, for radiative energy transport:

$$\frac{dT}{dr} = -\frac{3 \bar{\kappa} \rho}{4ac} \frac{L_r}{4\pi r^2} T^3$$

Convective Transport:

Convection is a three-dimensional process that must be approximated by one-dimensional equations for simple stellar interior models. It is a process that is difficult to model correctly, but begins with certain assumptions.

The pressure scale height H_p is defined as: $\frac{1}{H_p} \equiv \frac{-1}{P} \frac{dP}{dr}$

or, if $H_p = \text{constant}$, $P = P_0 e^{-r/H_p}$

H_p is the distance over which the gas pressure P decreases by a factor of $1/e$.

Stellar Models:

The complete set of differential equations describing the interiors of stars is therefore:

Equation of Continuity:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho$$

Hydrostatic Equilibrium:

$$\frac{dP}{dr} = \frac{-GM(r)\rho}{r^2}$$

Energy Generation:

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

Temperature Gradient:

$$\left(\frac{dT}{dr}\right)_{rad} = \frac{-3 \bar{\kappa} \rho}{4ac T^3} \frac{L_r}{4\pi r^2}$$

$$\left(\frac{dT}{dr}\right)_{ad} = \frac{-1}{C_P} \frac{GM(r)}{r^2}$$

At the “natural” boundaries of the star the corresponding values are:

– At the center: At the surface:

$$r = 0 \quad r = R$$

$$M(r) = 0 \quad M(r) = M_*$$

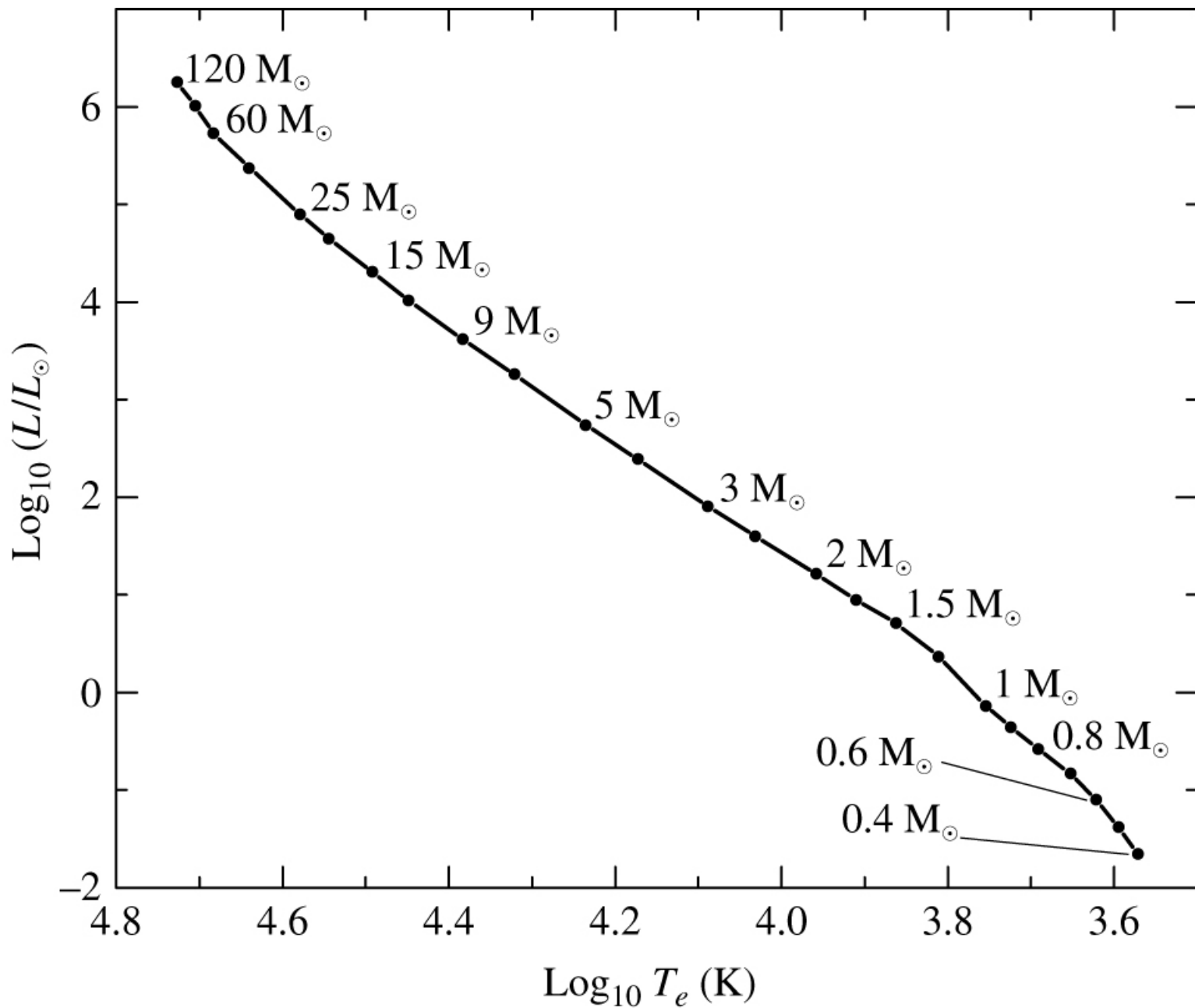
$$T(r) = T_c \quad T(r) = 0 \text{ (or } T_{\text{eff}})$$

$$P(r) = P_c \quad P(r) = 0$$

$$L(r) = L_c \quad L(r) = L_*$$

$$\rho(r) = \rho_c \quad \rho(r) = 0$$

Rotation and magnetic fields are usually ignored in most models (*i.e.* spherical symmetry is imposed), as well as any temporal changes (*i.e.* radial pulsation).



One of the basic tenets of stellar evolutionary models is the *Vogt-Russell Theorem*, which states that the mass and chemical composition of a star, and in particular how the chemical composition varies within the star, uniquely determine its radius, luminosity, and internal structure, as well as its subsequent evolution. A consequence of the theorem is that it is possible to uniquely describe all of the parameters for a star simply from its location in the *Hertzsprung-Russell Diagram*. There is no proof for the theorem, and in fact, it does fail in some special instances.

A prime example of where ambiguities arise occurs when one compares models for two stars, one of which is spherically symmetric and the other of which is flattened as a result of rapid rotation. Both stars can occupy the same location in the Hertzsprung-Russell diagram, at different evolutionary ages and even for different masses.