

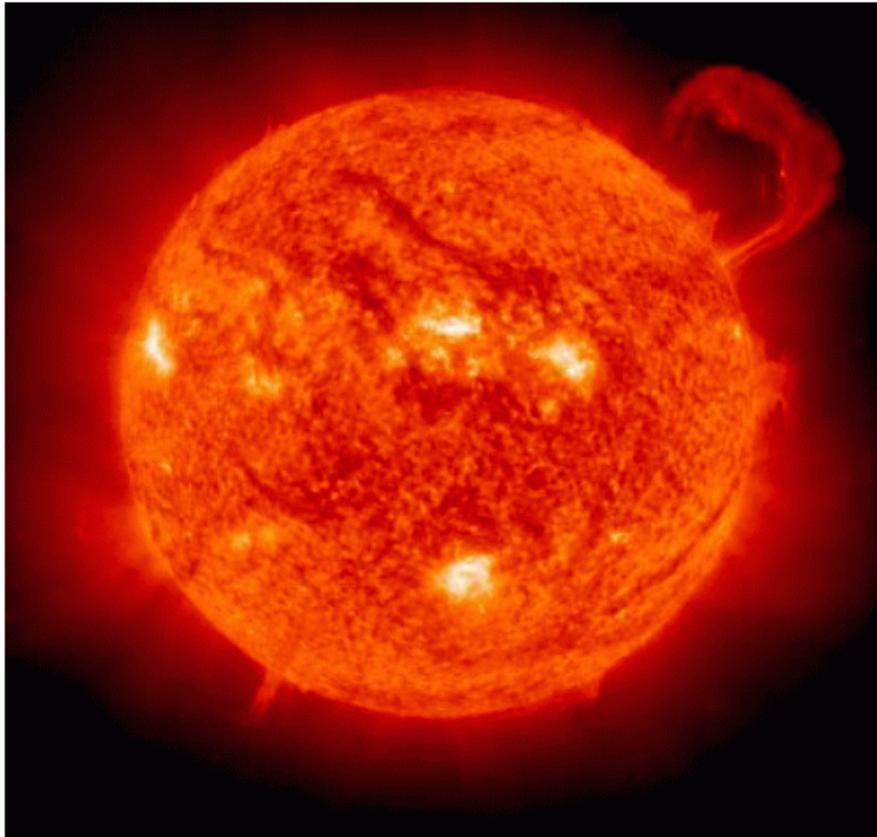
ASTR 400/700: Stellar Astrophysics

Stephen Kane



SAN FRANCISCO
STATE UNIVERSITY

Numbers you should know: The Sun



source: SOHO/EIT

Mass $\approx 2 \times 10^{30}$ kg = $1 M_{\odot}$

Radius $\approx 7 \times 10^8$ m = $1 R_{\odot}$

Distance = 1.5×10^{11} m = 1 AU

Luminosity = 4×10^{26} W = $1 L_{\odot}$

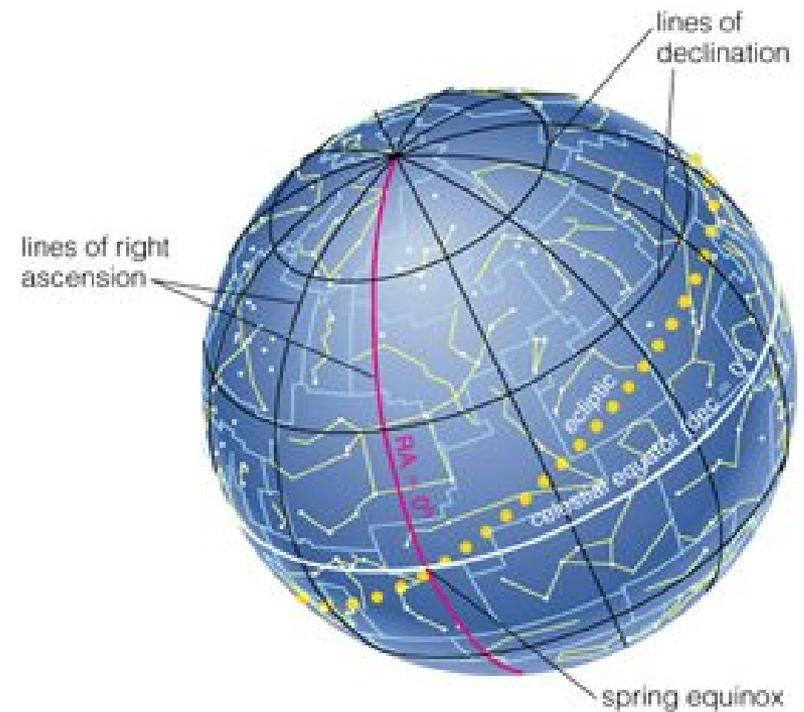
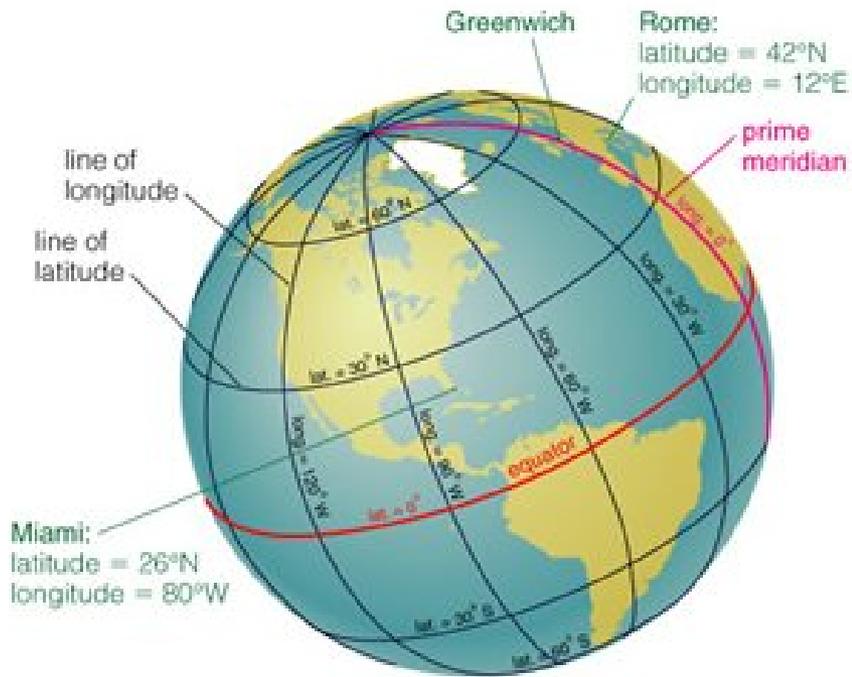
Surface temperature = 5800 K

age ≈ 4.5 Gyr

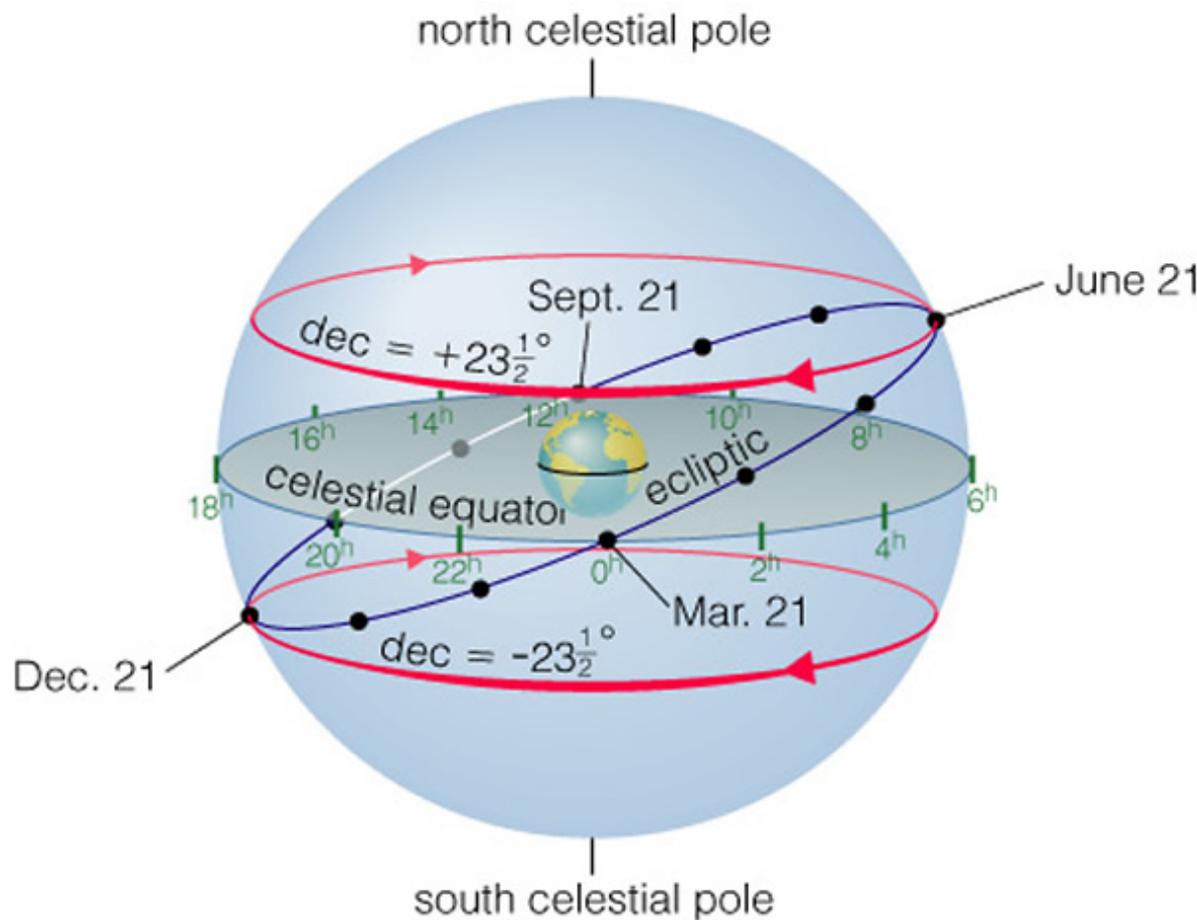
spectral type = G2 V

All other stars are scaled to these parameters for convenience.

Equatorial coordinate system



RA and Dec of the Cardinal Points on the Ecliptic



Vernal Equinox

- Sun appears on March 21
- RA = 0h Dec = 0°

Summer Solstice

- Sun appears on June 21
- RA = 6h Dec = 23.5°

Autumnal Equinox

- Sun appears on Sept. 21
- RA = 12h Dec = 0°

Winter Solstice

- Sun appears on Dec. 21
- RA = 18h Dec = -23.5°

Parallax and Distance

p = parallax angle

$$d \text{ (in parsecs)} = \frac{1}{p \text{ (in arcseconds)}}$$

$$d \text{ (in light-years)} = 3.26 \times \frac{1}{p \text{ (in arcseconds)}}$$

One **parsec** is the distance at which the mean radius of the Earth's orbit subtends an angle of one second of arc.

Flux, luminosity, and magnitude

$$F = \frac{L}{4\pi D^2}$$

$$m_2 - m_1 = 2.5 \log_{10} \left(\frac{F_1}{F_2} \right)$$

$$m_2 - m_1 = 2.5 \log_{10} \left(\frac{L_1}{4\pi D_1^2} \frac{4\pi D_2^2}{L_2} \right)$$

$$m_2 - m_1 = -2.5 \log_{10} \frac{L_2}{L_1} + 5 \log_{10} \frac{D_2}{D_1}$$

Absolute Magnitude and Distance Modulus

$m - M$ is a measure of the distance to a star and is called the **distance modulus**.

$$m - M = 5 \log_{10}(d) - 5 = 5 \log_{10} \left(\frac{d}{10 \text{ pc}} \right).$$

The absolute magnitude of the Sun is $M = 4.83$.

The luminosity of the Sun is $L = 3.846 \times 10^{26} \text{ W}$

$$M = M_{\text{Sun}} - 2.5 \log_{10} \left(\frac{L}{L_{\odot}} \right),$$

Note the M includes only light in the visible band, so this is accurate only for stars with the same spectrum as the Sun.

Stefan-Boltzmann law

- Stefan-Boltzmann constant:

$$\sigma = 5.670400 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}.$$

- For a spherical star of radius R:

$$L = 4\pi R^2 \sigma T_e^4.$$

- **The Stefan-Boltzmann equation.**

Wien's law

- Cooler objects produce radiation which peaks at lower energies = longer wavelengths = redder colors.
- Hotter objects produce radiation which peaks at higher energies = shorter wavelengths = bluer colors.
- Wavelength of peak radiation:
Wien's Displacement Law

$$\lambda_{\max} T = 0.002897755 \text{ m K.}$$

Planck Law

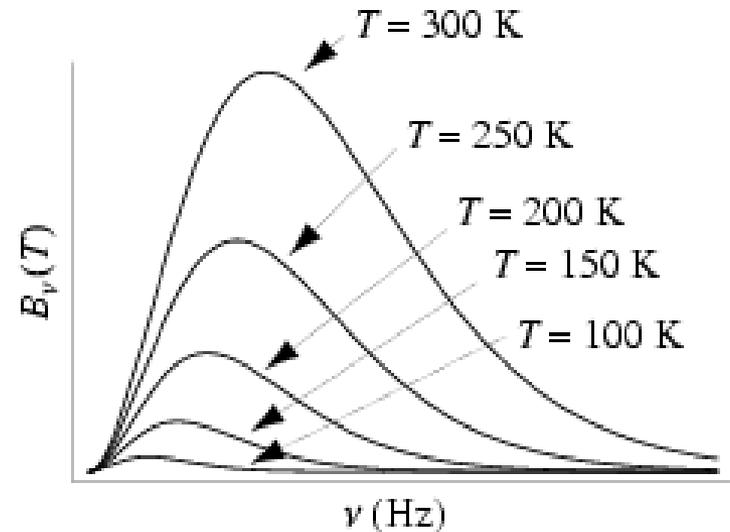
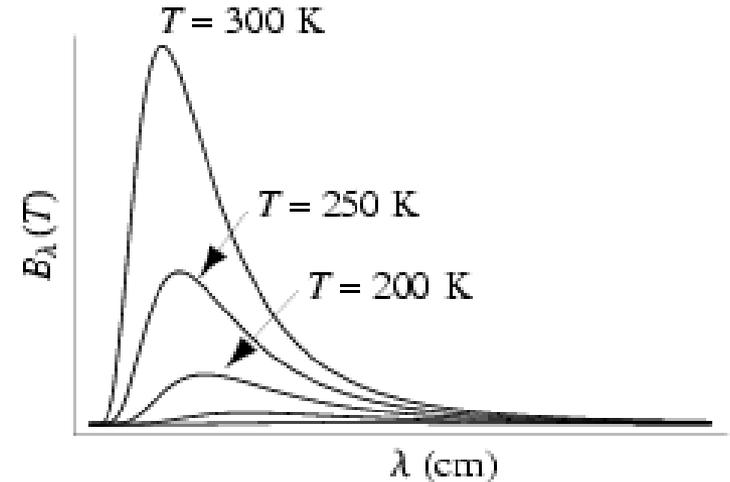
- We have two forms. As a function of wavelength.

$$B_{\lambda}(T) = \frac{2hc^2/\lambda^5}{e^{hc/\lambda kT} - 1}$$

And as a function of frequency

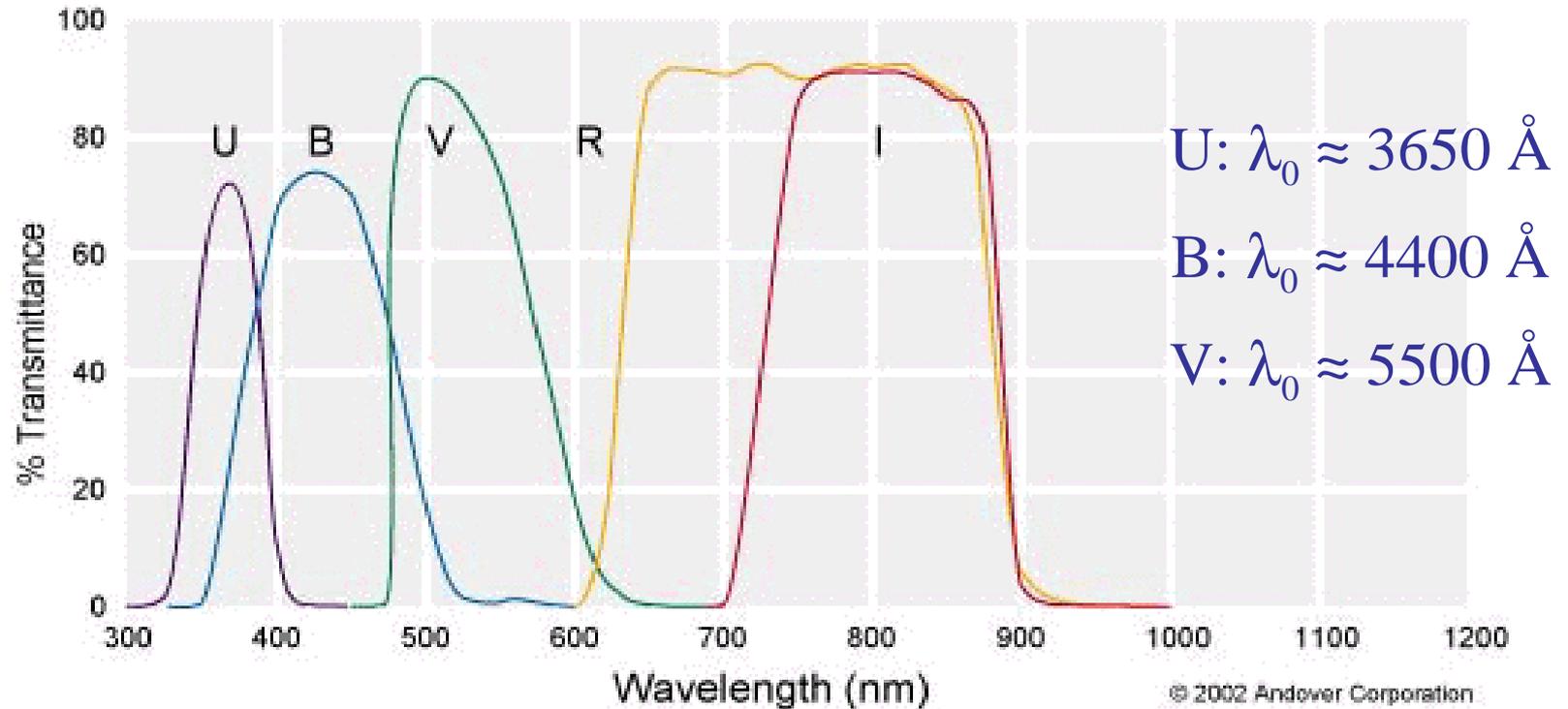
$$B_{\nu}(T) = \frac{2h\nu^3/c^2}{e^{h\nu/kT} - 1}$$

The Planck Law gives a distribution that peaks at a certain wavelength, the peak shifts to shorter wavelengths for higher temperatures, and the area under the curve grows rapidly with increasing temperature.



Optical Wavelength Bands

Kron/Cousins UBVRI Filters



- U filter (P/N KRON-U-XX)
- B filter (P/N KRON-B-XX)
- V filter (P/N KRON-V-XX)
- R filter (P/N KRON-R-XX)
- I filter (P/N KRON-I-XX)

The Color Index

We define the **Color Index**

$$B - V$$

(i.e., B magnitude – V magnitude)

The **bluer** a star appears, the **smaller** the color index $B - V$.

The **hotter** a star is, the **smaller** its color index $B - V$.



$B - V$ 

 Temperature

Kirchoff's laws

Chemical Analysis by Spectral Observations (Kirchoff & Bunsen)

- A hot solid, liquid, or dense gas produces a continuous spectrum.
- A thin gas in front of a cooler background produces an emission line spectrum.
- A thin gas in front of a hot source imprints absorption lines on the spectrum. This is mainly what we see from stars.

So when the electron is in any energy level n :

$$r_n = \frac{4\pi\epsilon_0\hbar^2}{\mu e^2} n^2 = a_0 n^2,$$

$$E_n = -\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2} \frac{1}{n^2} = -13.6 \text{ eV} \frac{1}{n^2}.$$

Bohr radius $a_0 = \epsilon_0\hbar^2 / \pi m e^2 = 5.29 \times 10^{-11} \text{ m}$

Then the energy of an emitted photon is:

$$E_{\text{photon}} = E_{\text{high}} - E_{\text{low}}$$

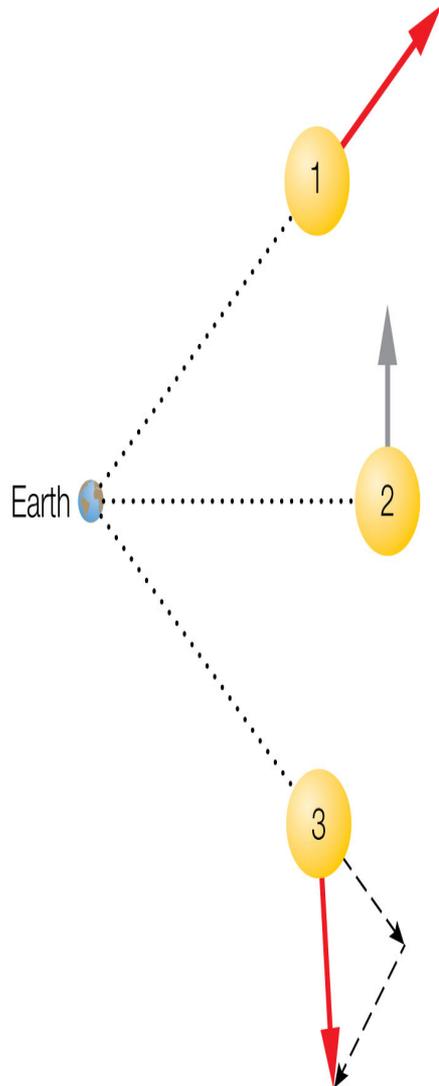
$$\frac{hc}{\lambda} = \left(-\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2} \frac{1}{n_{\text{high}}^2} \right) - \left(-\frac{\mu e^4}{32\pi^2\epsilon_0^2\hbar^2} \frac{1}{n_{\text{low}}^2} \right)$$

$$\frac{1}{\lambda} = \frac{\mu e^4}{64\pi^3\epsilon_0^2\hbar^3 c} \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2} \right)$$

Explaining Kirchoff's laws

- A hot solid, liquid, or dense gas produces a continuous spectrum. **Blackbody radiation described by the Planck function and Wien's law.**
- A thin gas in front of a cooler background produces an emission line spectrum. **Downward transition of electron producing a single photon.**
- A thin gas in front of a hot source imprints absorption lines on the spectrum. This is mainly what we see from stars. **Upward transition of electron depending on energy of incident photon.**

Doppler shift tells us **ONLY** about the part of an object's motion toward or away from us.



Radial velocity motion

$$\frac{\lambda_{\text{obs}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{\Delta\lambda}{\lambda_{\text{rest}}} = \frac{v_r}{c},$$

Transverse (proper) motion

Combined motion

Table 11.1 *The Spectral Sequence*

Spectral Type	Example(s)	Temperature Range	Key Absorption Line Features	Brightest Wavelength (color)	Typical Spectrum
O	Stars of Orion's Belt	>30,000 K	Lines of ionized helium, weak hydrogen lines	>97 nm (ultraviolet)*	
B	Rigel	30,000 K–10,000 K	Lines of neutral helium, moderate hydrogen lines	97–290 nm (ultraviolet)*	
A	Sirius	10,000 K–7500 K	Very strong hydrogen lines	290–390 nm (violet)*	
F	Polaris	7500 K–6000 K	Moderate hydrogen lines, moderate lines of ionized calcium	390–480 nm (blue)*	
G	Sun, Alpha Centauri A	6000 K–5000 K	Weak hydrogen lines, strong lines of ionized calcium	480–580 nm (yellow)	
K	Arcturus	5000 K–3500 K	Lines of neutral and singly ionized metals, some molecules	580–830 nm (red)	
M	Betelgeuse, Proxima Centauri	<3500 K	Strong molecular lines	> 830 nm (infrared)	

*All stars above 6000 K look more or less white to the human eye because they emit plenty of radiation at all visible wavelengths.

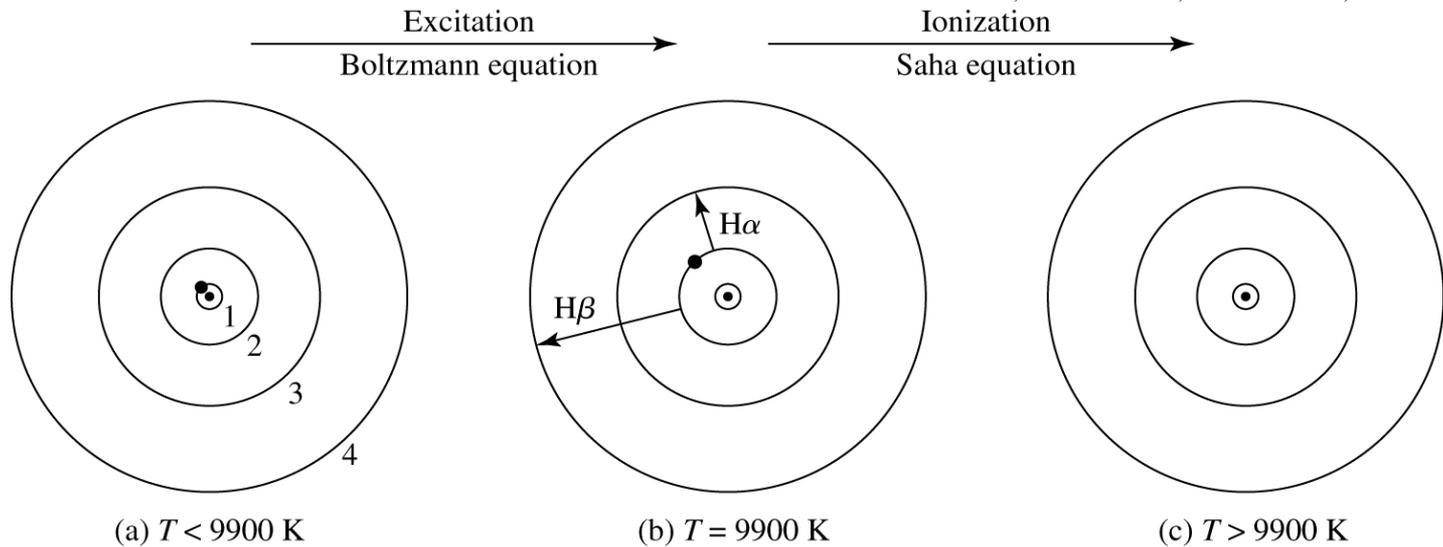
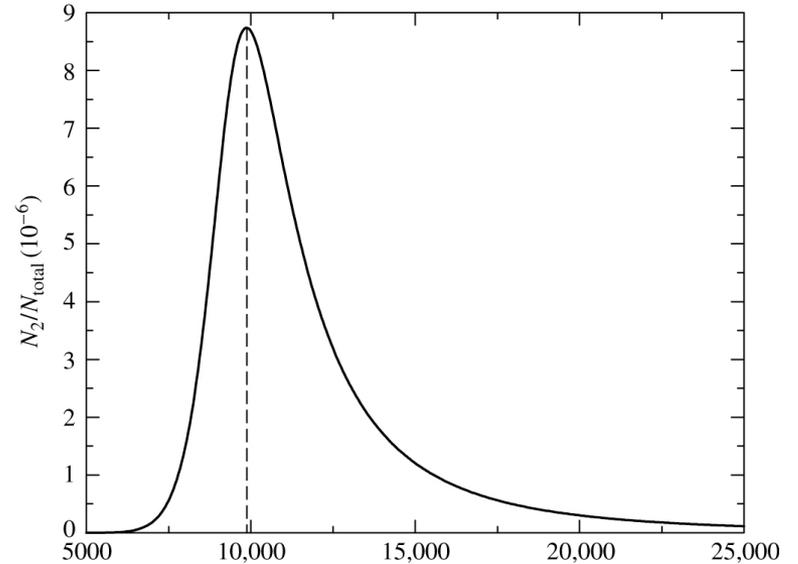
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Lines in a star's spectrum correspond to a *spectral type* that reveals its temperature:

(Hottest) O B A F G K M (Coolest)

Number of Excited Hydrogen Atoms

- Convolution of Boltzmann and Saha Equations
- Maximum occurs at 9500K due to lack of un-ionized atoms above this temperature



By carefully examining a star's spectral lines, astronomers can determine whether that star is a main-sequence star, giant, supergiant, or white dwarf

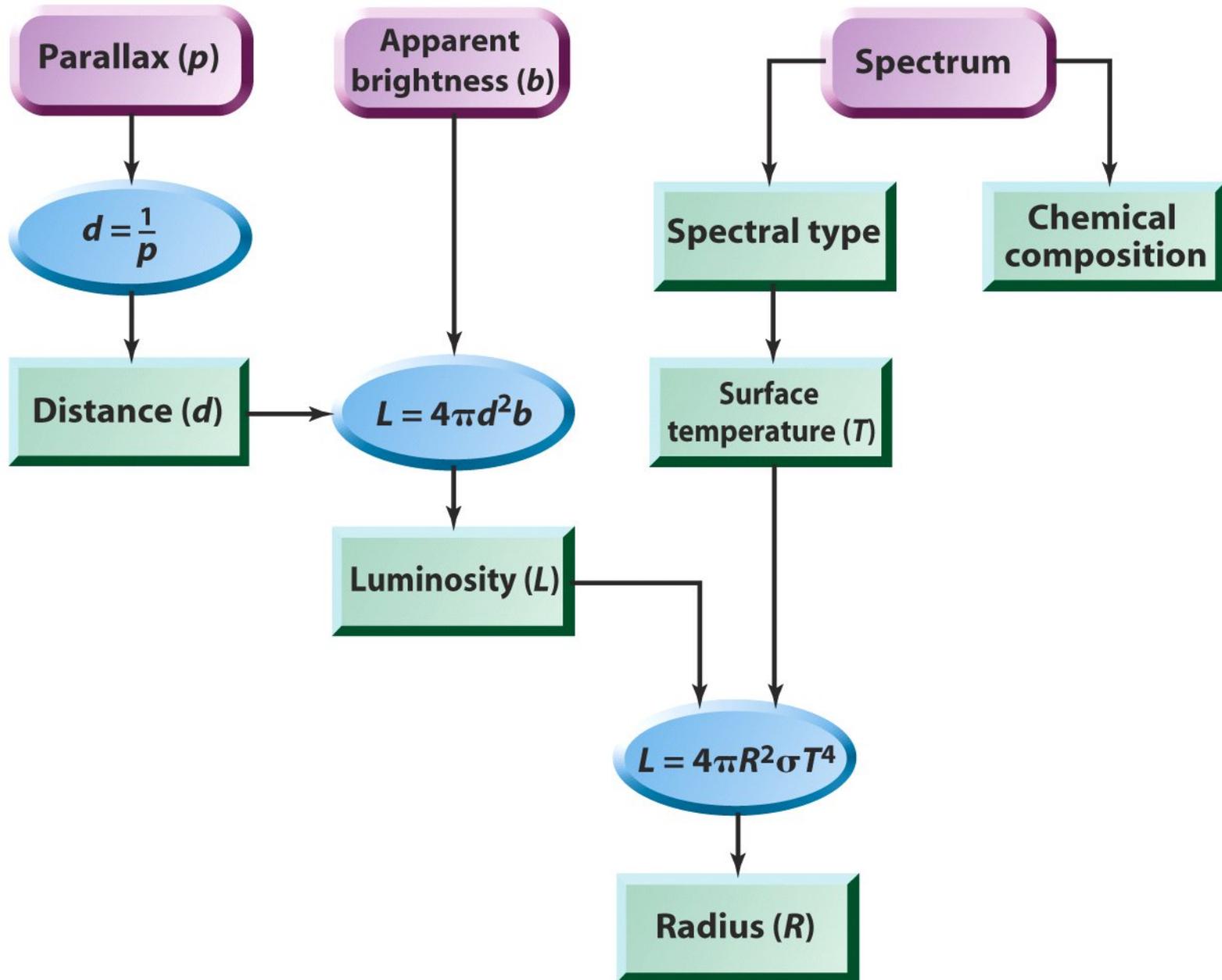


(a) A supergiant star has a low-density, low-pressure atmosphere: its spectrum has narrow absorption lines



(b) A main-sequence star has a denser, higher-pressure atmosphere: its spectrum has broad absorption lines

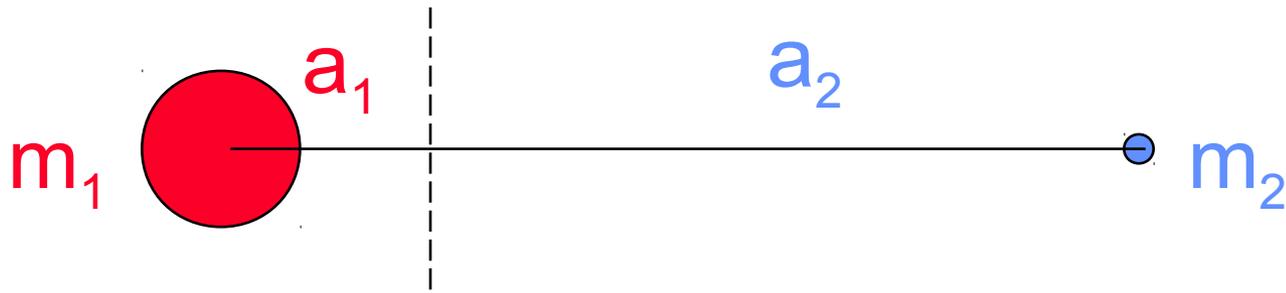
Flowchart of Key Stellar Parameters



Types of Binary Star Systems

- Optical double
- Visual binary
- Astrometric binary
- Eclipsing binary
- Spectrum binary
- Spectroscopic binary

About half of all stars are in binary systems.



Centre of mass of
binary system

Distance is related to the angular semi-major axes.

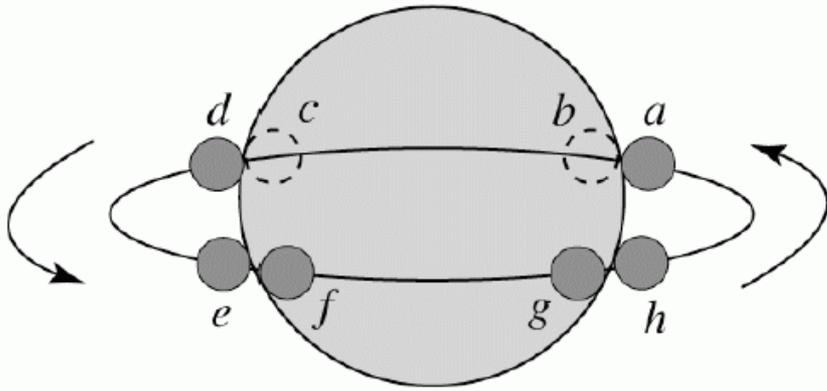
$$\alpha_1 = \frac{a_1}{d} \quad \text{and} \quad \alpha_2 = \frac{a_2}{d},$$

$$\frac{m_1}{m_2} = \frac{\alpha_2}{\alpha_1}.$$

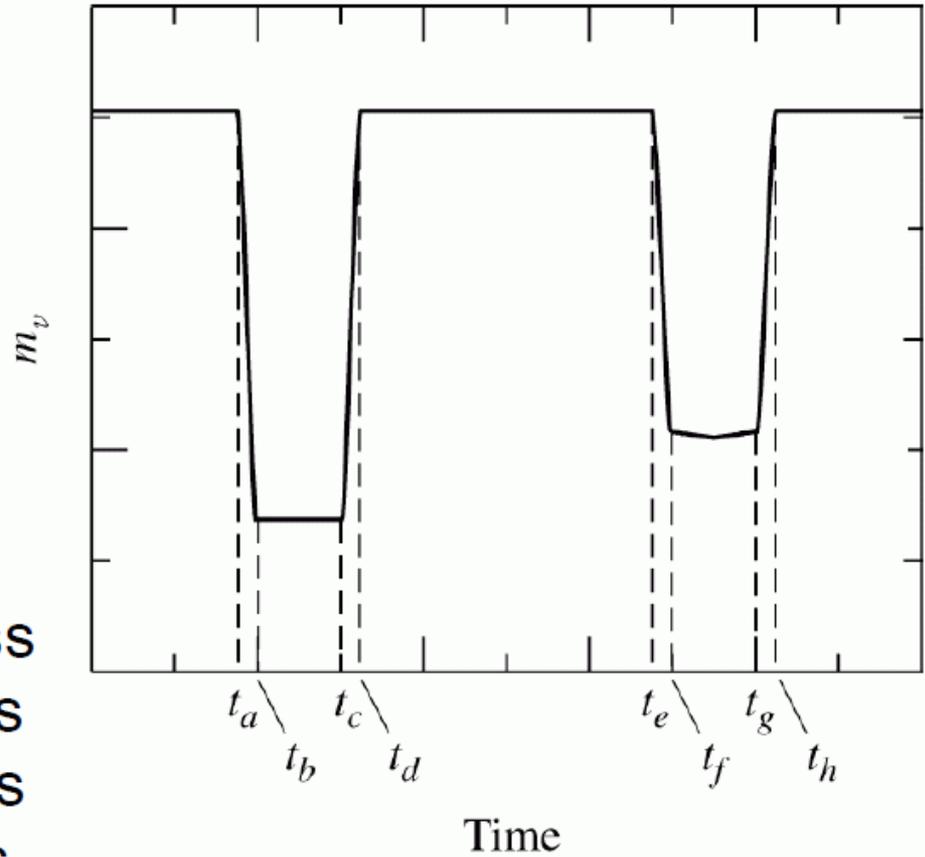
Using Kepler's third law we determine the sum of the masses.

$$m_1 + m_2 = \frac{4\pi^2 (\alpha d)^3}{G P^2} = \frac{4\pi^2}{G} \left(\frac{d}{\cos i} \right)^3 \frac{\tilde{\alpha}^3}{P^2},$$

Totally Eclipsing Binaries



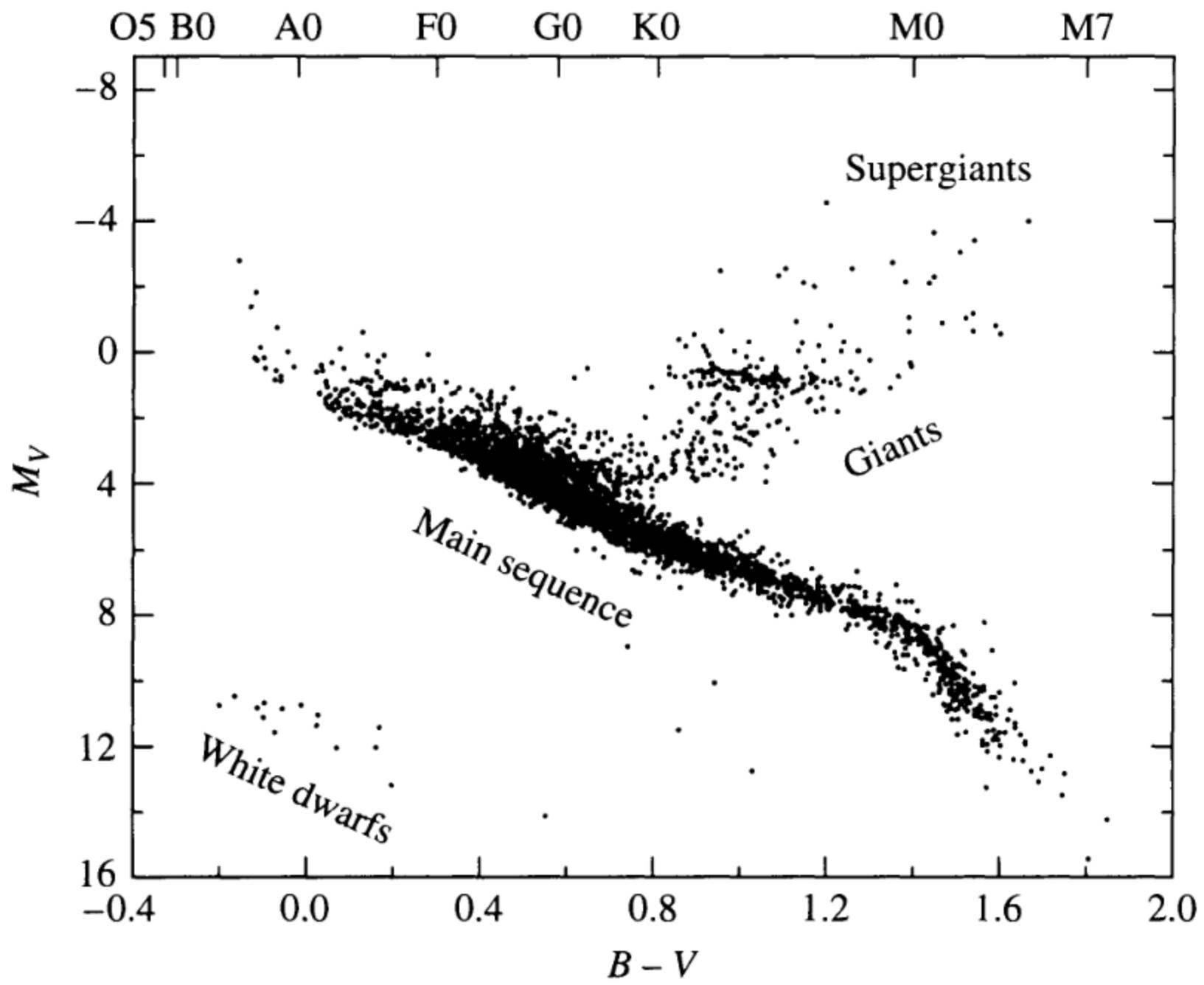
t_a – start of secondary ingress
 t_b – end of secondary ingress
 t_c – start of secondary egress
 t_d – end of secondary egress



The story so far

- Distances
- Radial velocity
- Proper motion & tangential velocity
- Flux – distance – luminosity
- Apparent magnitudes
- Absolute magnitudes
- Spectral types
- Ionization vs temperature
- Diameters of stars
- Masses of stars
- Spectroscopic binaries
- Mass-luminosity relationship

The Hertzsprung-Russell diagram can visualize all of these things



Average density of stars:

The Sun (G2V):

$$\bar{\rho} = \frac{M_{sun}}{\frac{4}{3}\pi R_{sun}^3} = 1.4 \text{ gcm}^{-3}$$

Sirius (A1V):

$$\bar{\rho} = \frac{M_{Sirius}}{\frac{4}{3}\pi R_{Sirius}^3} = 0.76 \text{ gcm}^{-3}$$

Betelgeuse (M2I):

$$\bar{\rho} = \frac{M_{Bet}}{\frac{4}{3}\pi R_{Bet}^3} = 10^{-11} \text{ gcm}^{-3}$$

Betelgeuse has an average density that is 100,000 times less dense than the air we breathe!



Airy rings around a single star.



The interference pattern produced as the Airy rings of two stars overlap each other.

No image can be smaller than the innermost Airy ring, and this sets the **diffraction limit** of a telescope to be

$$\theta_{\min} = 1.22(206,265) \frac{\lambda}{D}$$

where D is the telescope's **aperture**

= diameter of the telescope's mirror

(in same units used to measure the wavelength).

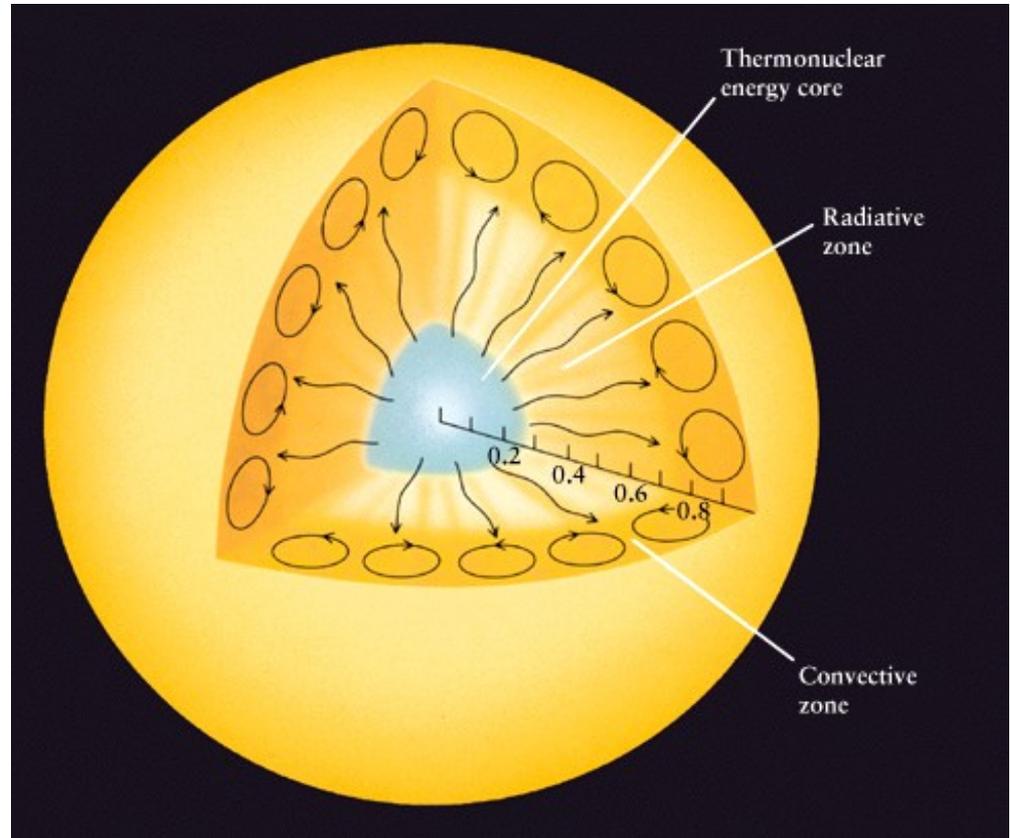
θ_{\min} (the above expression gives the answer in arcseconds) sets a limit to the **resolution** of the image.

Overview

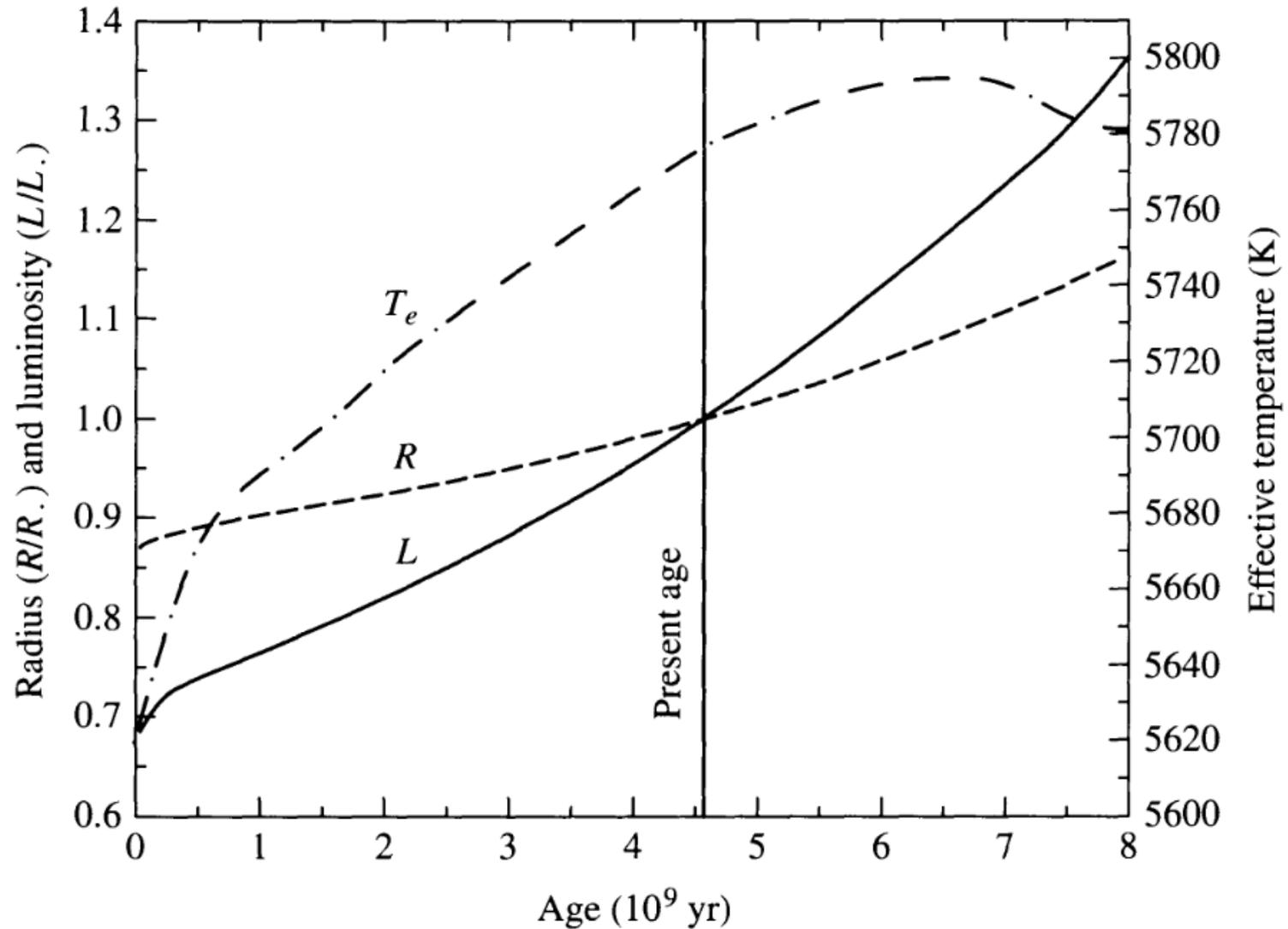
- Spectrum, spectral resolution
- Dispersion (prism, grating)
- Spectrographs
 - longslit
 - echelle
 - fourier transform
- Multiple Object Spectroscopy

The Solar Interior - “The Standard Model”

- Core
 - Energy generated by nuclear fusion (the proton-proton chain).
- Radiative Zone
 - Energy transport by radiation.
- Convective Zone
 - Energy transport by convection.



The Solar Constant



Definition of Opacity

- Consider a beam of parallel light rays traveling through a gas.
- Any process that removes photons from this beam of light is called **absorption**
- **Absorption** includes **Scattering!!**
- True absorption is by electronic transitions in atoms (and sometimes molecules)
 - Change in Intensity is

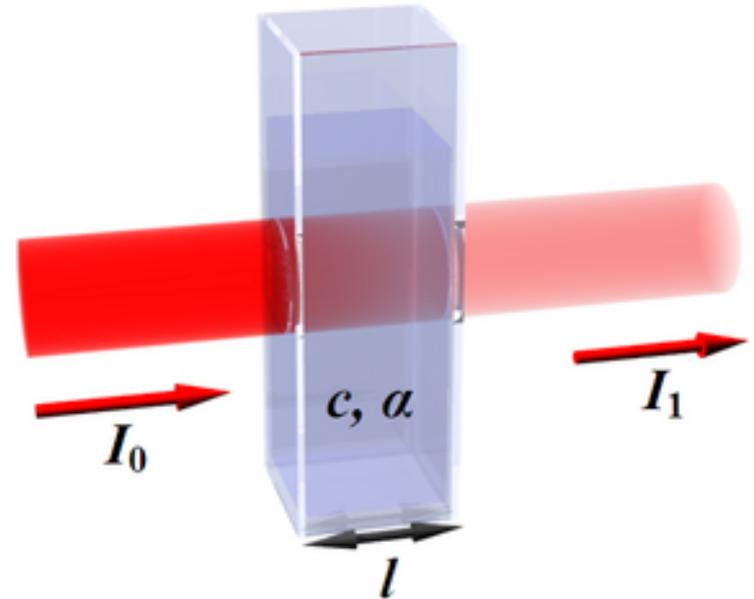
$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds.$$

Proportional to:

distance traveled

density of gas

absorption coefficient



Optical Depth

- Unit-less Measure of amount of attenuation.
- Accounts for varying density

$$[\text{density}] = \text{kg/m}^3$$

$$[\text{opacity}] = \text{m}^2/\text{kg}$$

$$[\text{Optical Depth}] = [\text{density}][\text{opacity}][\text{distance}] = 1$$

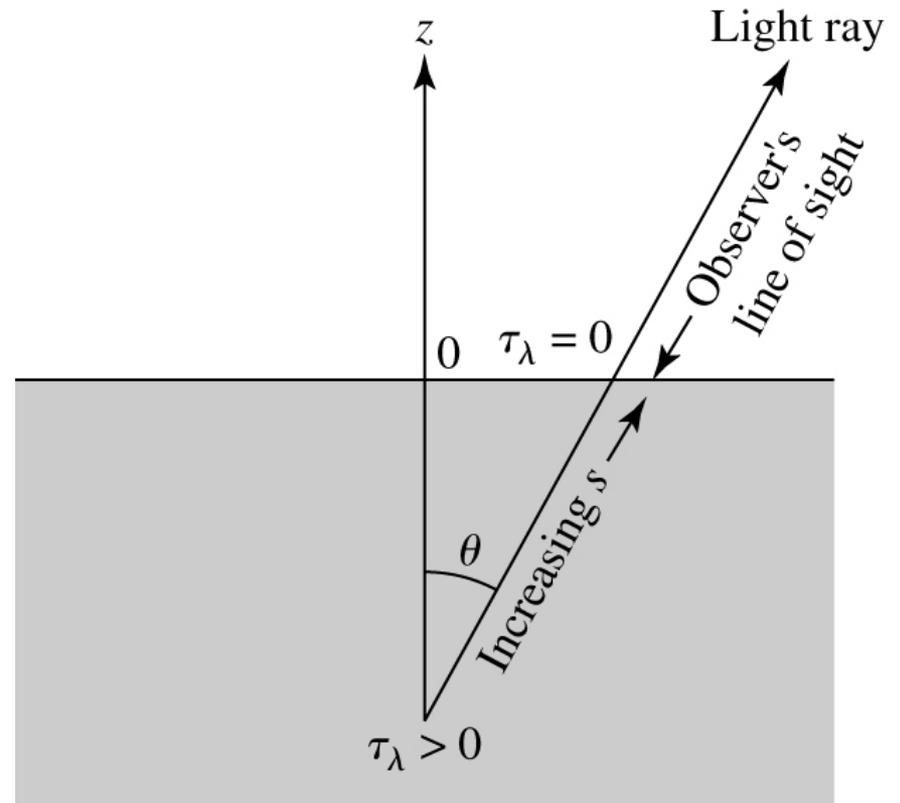
$$l = \frac{1}{\kappa_\lambda \rho} = \frac{1}{n\sigma_\lambda}$$

$$d\tau_\lambda = -\kappa_\lambda \rho ds,$$

$$\Delta\tau_\lambda = \tau_{\lambda,f} - \tau_{\lambda,0} = -\int_0^s \kappa_\lambda \rho ds.$$

$$0 - \tau_{\lambda,0} = -\int_0^s \kappa_\lambda \rho ds$$

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho ds.$$



$$I_\lambda = I_{\lambda,0} e^{-\tau_\lambda}.$$

Rosseland Mean Opacity

- An attempt at estimating the average opacity over all wavelengths
- Weight by the rate at which Intensity distribution (blackbody radiation) varies with temperature.
- Determine dependence of other parameters such as temperature

$$X \equiv \frac{\text{total mass of hydrogen}}{\text{total mass of gas}}$$

$$Y \equiv \frac{\text{total mass of helium}}{\text{total mass of gas}}$$

$$Z \equiv \frac{\text{total mass of metals}}{\text{total mass of gas}}$$

$$\frac{1}{\bar{\kappa}} \equiv \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu(T)}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu(T)}{\partial T} d\nu}$$

$$\bar{\kappa}_{\text{bf}} = 4.34 \times 10^{21} \frac{g_{\text{bf}}}{t} Z(1+X) \frac{\rho}{T^{3.5}} \text{ m}^2 \text{ kg}^{-1}$$

$$\bar{\kappa}_{\text{ff}} = 3.68 \times 10^{18} g_{\text{ff}} (1-Z)(1+X) \frac{\rho}{T^{3.5}} \text{ m}^2 \text{ kg}^{-1},$$

$$\bar{\kappa}_{\text{es}} = 0.02(1+X) \text{ m}^2 \text{ kg}^{-1}.$$

$$\bar{\kappa}_{\text{H}^-} \approx 7.9 \times 10^{-34} (Z/0.02) \rho^{1/2} T^9 \text{ m}^2 \text{ kg}^{-1}.$$

$$\bar{\kappa} = \kappa_{\text{bb}} + \kappa_{\text{bf}} + \kappa_{\text{ff}} + \kappa_{\text{es}} + \kappa_{\text{H}^-}.$$

How far do we look into a star?

- Looking into a star at any angle, we always look back to an optical depth of about $\tau_\lambda = 2/3$ as measured straight back along the line of sight
- Photon's at a distance of less than 1 mean free path from the surface are likely to escape
- Star's photosphere is defined to be the layer from which visible light originates
 - Formation of spectral lines (absorption) occur because temperature of the material in the star decreases outwards from the center of the star

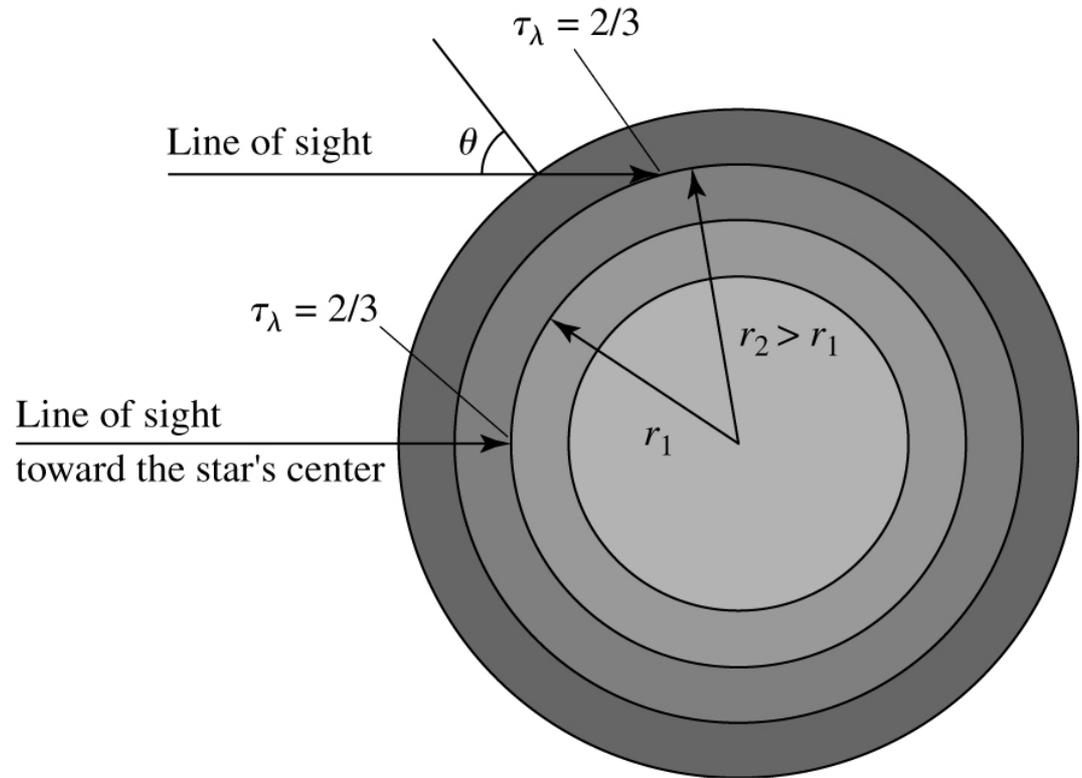


Limb Darkening

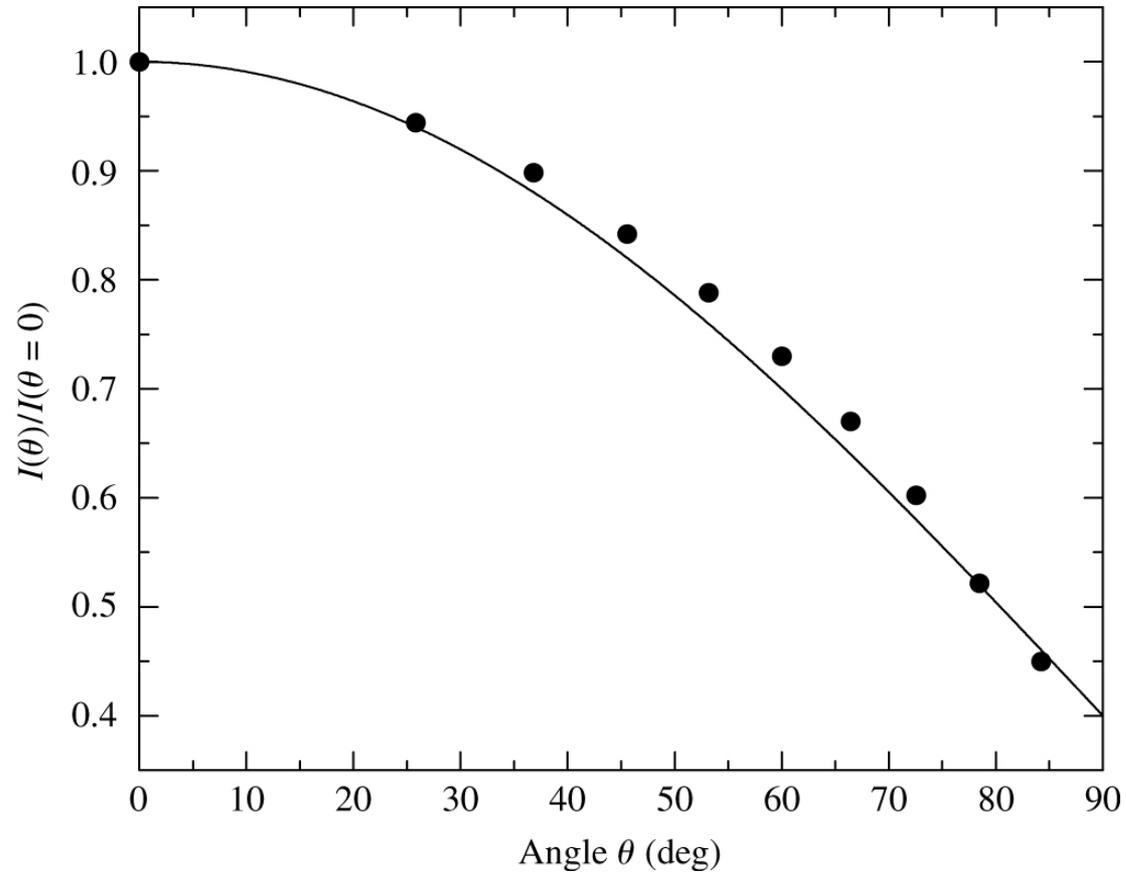
- When looking at the center of the sun one can see “deeper” than when looking at the edge of the sun
 - Deeper is hotter
 - Hotter is brighter...



LIMB DARKENING



Eddington Approximation of Solar Limb Darkening



Profiles of Spectral Lines

- Spectral Line conveys information about the environment in which it was formed

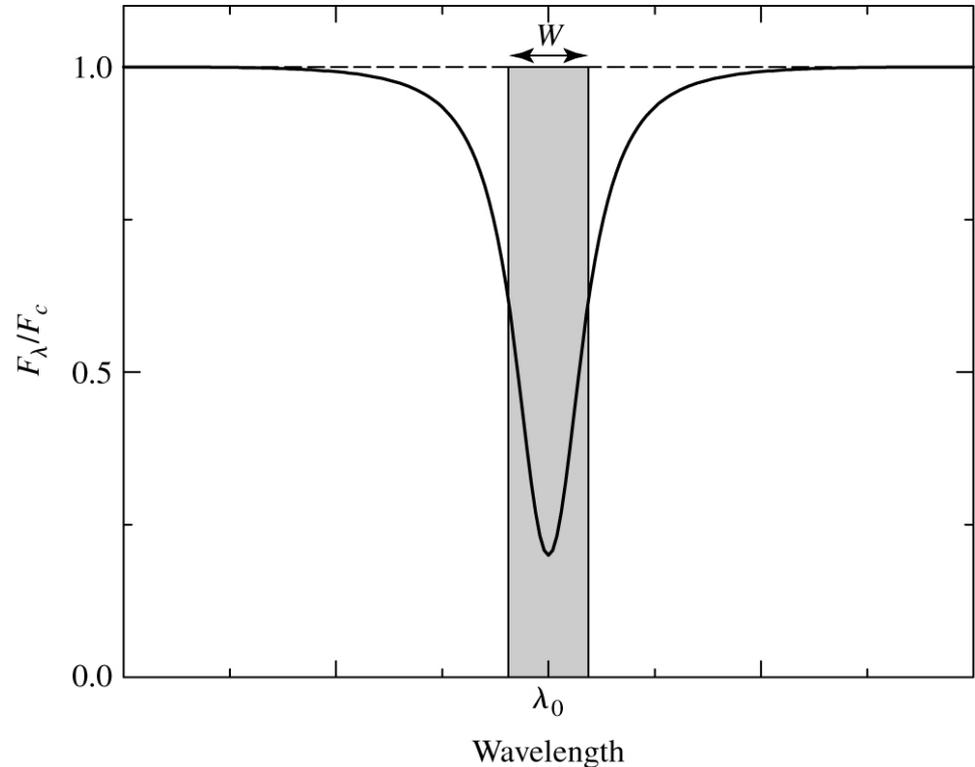
- Line width

$$W = \int \frac{F_c - F_\lambda}{F_c} d\lambda,$$

- Line Shape

– Broadening

– Profile



Optically Thin Spectral Line. Thus termed because there is no wavelength at which the radiant flux has been **completely** blocked.

Sources of Broadening

- **Natural Broadening**

- From Uncertainty Principle
- $\sim 2-4 \times 10^{-5}$ nm

- **Doppler Broadening**

- From thermal motion of atoms
- $\sim 4.27 \times 10^{-2}$ nm at 5772K

- **Pressure and Collisional Broadening**

- From atomic orbitals being perturbed from collisions
- $\sim 2-4 \times 10^{-5}$ nm

Stellar Structure

Stellar evolution results from a constant battle with gravity!

- Variation of temperature
- Variation of density
- Variation of pressure

Test against observations with computer models.

Physical Principles

- Hydrostatic Equilibrium
- Virial Theorem
- Energy Generation
- Energy Transport

Models of Stars

The parameters used for studying and modeling stellar interiors include:

r = radial distance from the centre of the star

$M(r)$ = mass interior to r

$T(r)$ = temperature at r

$P(r)$ = pressure at r

$L(r)$ = luminosity at r

$\varepsilon(r)$ = energy generation at r

$\kappa(r)$ = opacity at r

$\rho(r)$ = density at r

In modern models mass M is used as the dependent variable rather than radial distance r , but it is more informative to initiate the study of stellar interiors using the geometrical variable r .

Stellar Energy Sources

The gravitational potential energy required to contract a star to its present size is given by:

$$V = -\frac{3}{5} \frac{GM^2}{R}$$

But, of the potential energy lost by a star, according to the Virial Theorem, one half is transformed into an increase in the kinetic energy of the gas (heat) and the remainder is radiated into space. The radiation lost by a star upon contraction to the main sequence is therefore given by:

$$E = +\frac{3}{10} \frac{GM^2}{R}$$

For the Sun, at its present mass (1.9891×10^{33} gm) and radius (6.9598×10^{10} cm), the amount of energy radiated through contraction is:

$$E = +\frac{3}{10} \frac{(6.672 \times 10^{-8})(1.9891 \times 10^{33})^2}{(6.9598 \times 10^{10})} = 1.138 \times 10^{48} \text{ ergs}$$

Stellar Models:

The complete set of differential equations describing the interiors of stars is therefore:

Equation of Continuity:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho$$

Hydrostatic Equilibrium:

$$\frac{dP}{dr} = \frac{-GM(r)\rho}{r^2}$$

Energy Generation:

$$\frac{dL}{dr} = 4\pi r^2 \rho \varepsilon$$

Temperature Gradient:

$$\left(\frac{dT}{dr}\right)_{rad} = \frac{-3 \bar{\kappa} \rho}{4ac T^3} \frac{L_r}{4\pi r^2}$$

$$\left(\frac{dT}{dr}\right)_{ad} = \frac{-1}{C_P} \frac{GM(r)}{r^2}$$

At the “natural” boundaries of the star the corresponding values are:

– At the centre: At the surface:

$$r = 0 \quad r = R$$

$$M(r) = 0 \quad M(r) = M_*$$

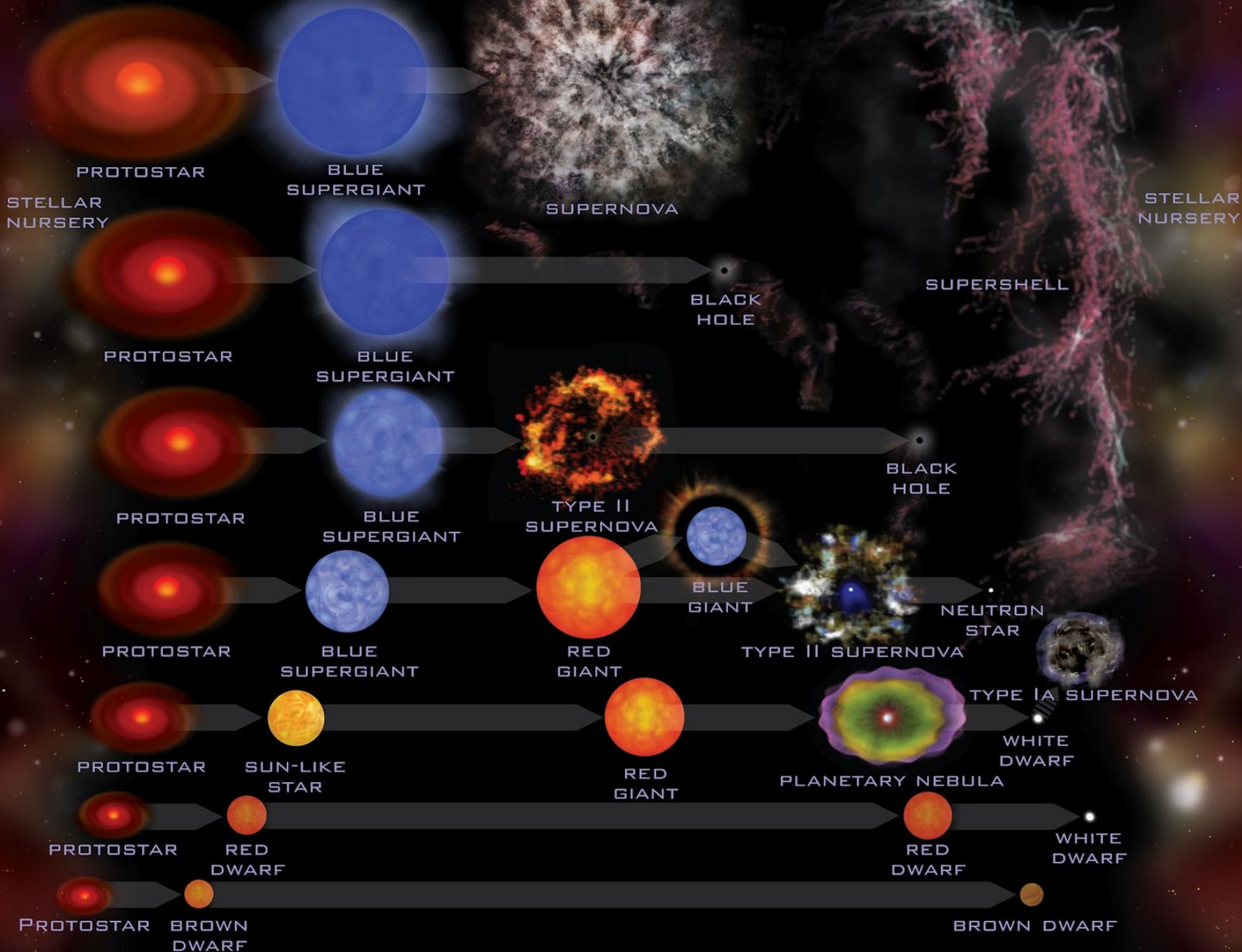
$$T(r) = T_c \quad T(r) = 0 \text{ (or } T_{\text{eff}})$$

$$P(r) = P_c \quad P(r) = 0$$

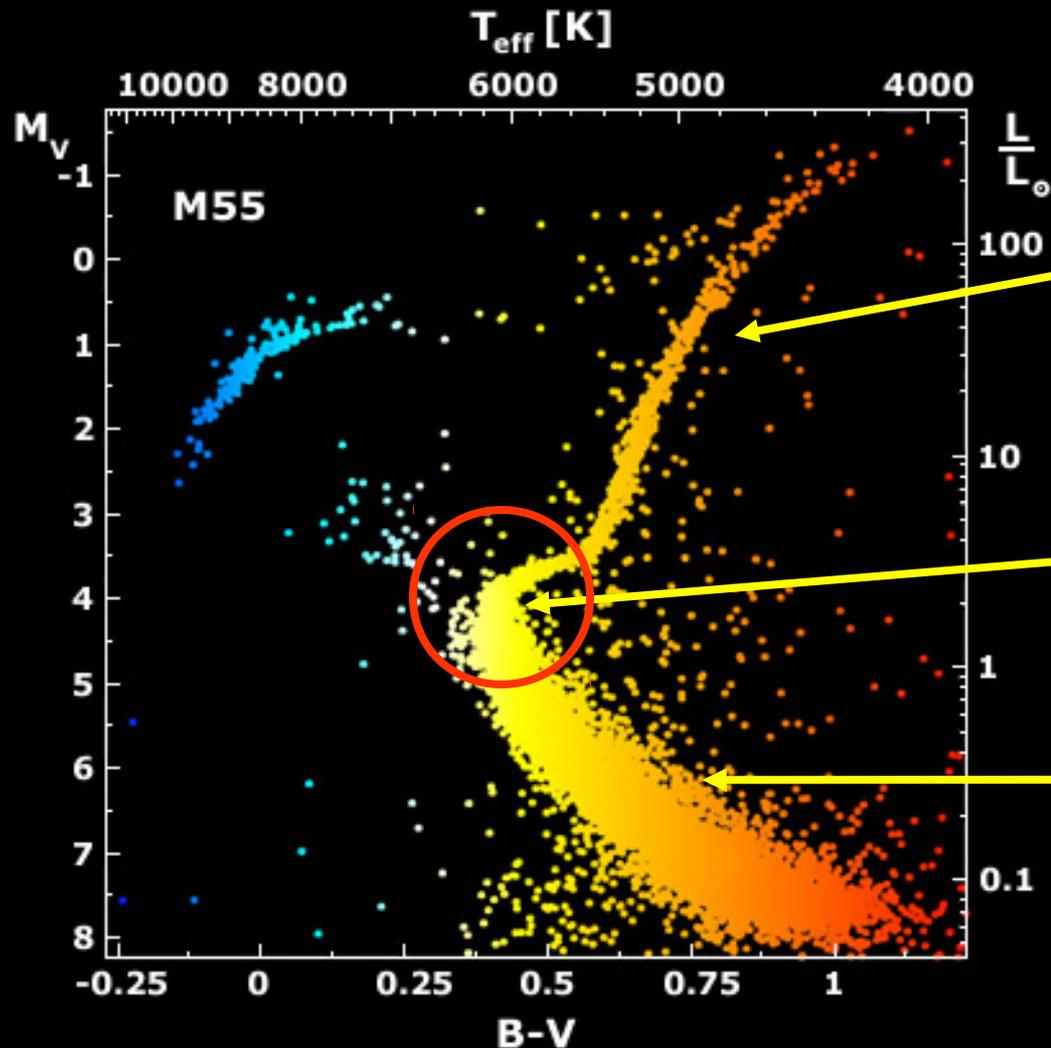
$$L(r) = L_c \quad L(r) = L_*$$

$$\rho(r) = \rho_c \quad \rho(r) = 0$$

Rotation and magnetic fields are usually ignored in most models (*i.e.* spherical symmetry is imposed), as well as any temporal changes (*i.e.* radial pulsation).



Evidence for Stellar Evolution: HR Diagram of the Star Cluster M 55

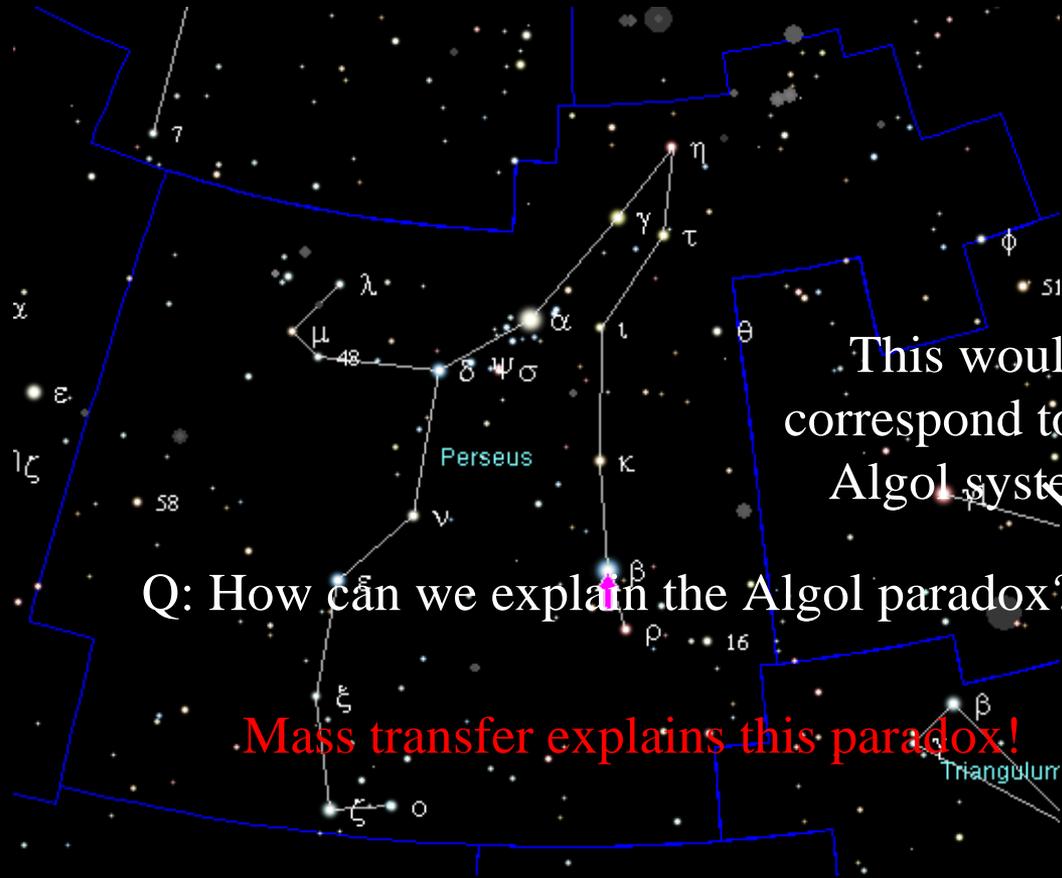


High-mass stars evolved onto the giant branch

Turn-off point

Low-mass stars still on the main sequence

The "Algol paradox"

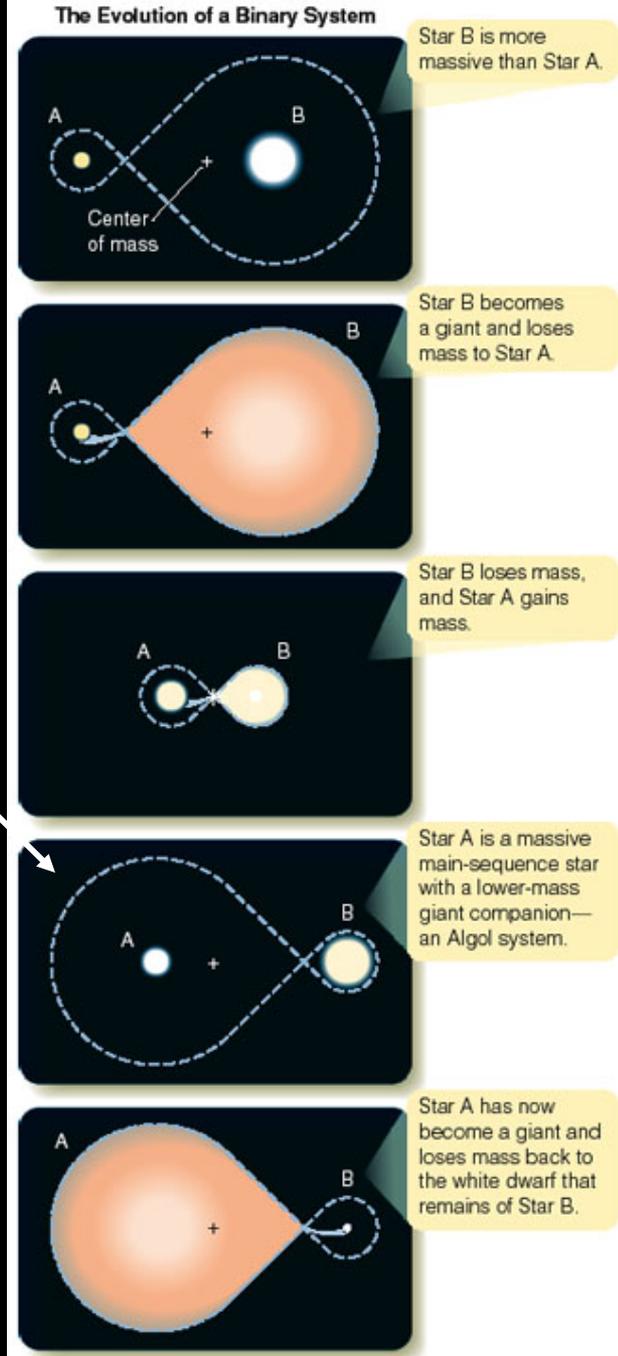


This would correspond to the Algol system

Q: How can we explain the Algol paradox?

Mass transfer explains this paradox!

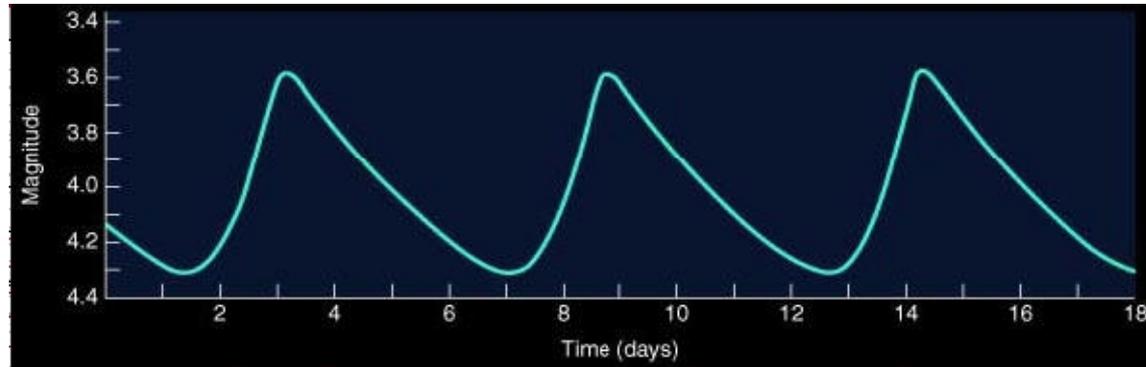
The less massive star became a giant while the more massive star remained on the main-sequence!?! $\tau = \frac{1}{M^{2.5}}$



Delta-Cephei

John Goodricke discovered in 1784 that the brightness of Delta-Cephei was variable with a period of about 5 days!!!!

Prototype of the classical **cepheid** variable star

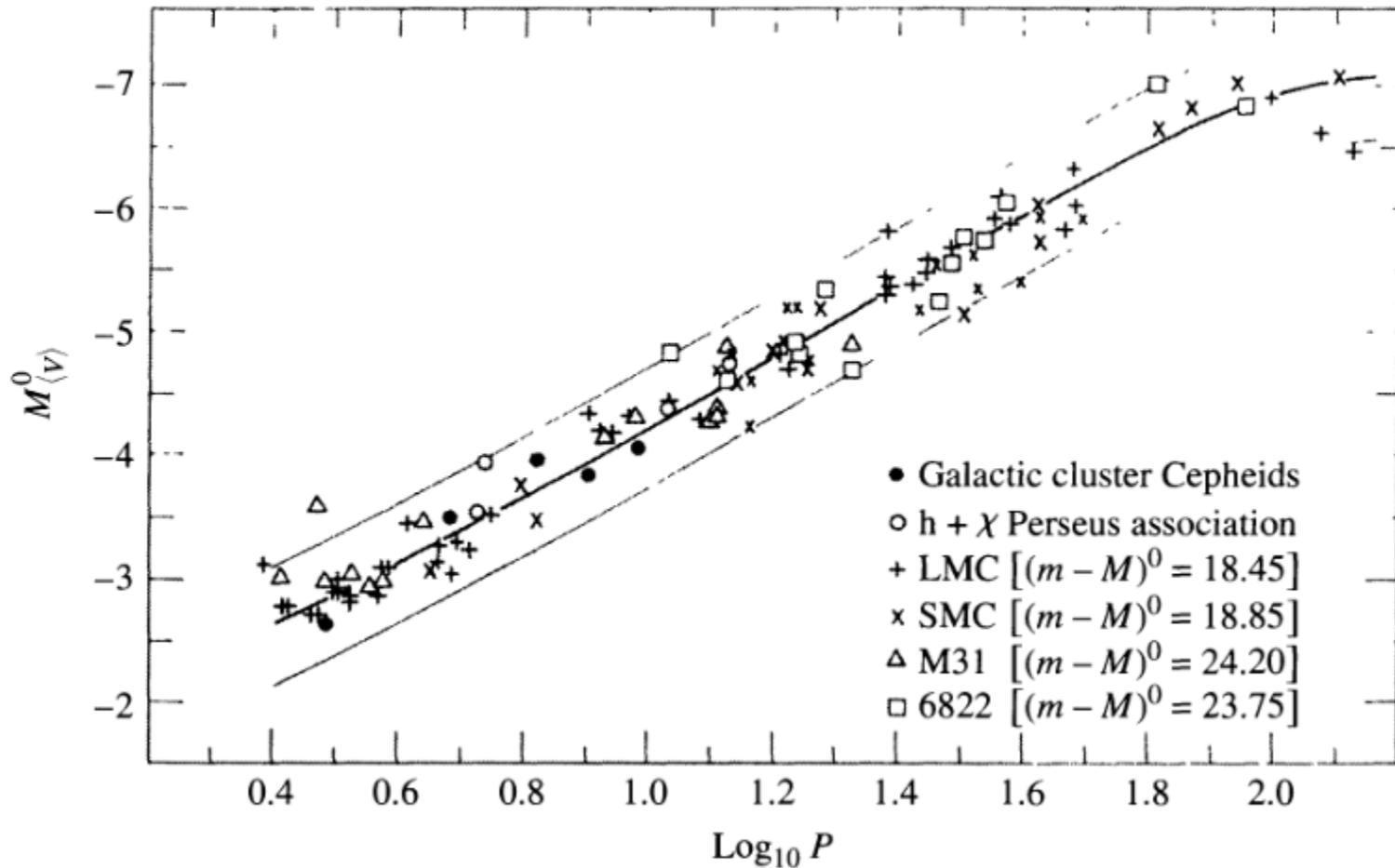


magnitude varies from 3.4 to 4.3,
→ luminosity changes by factor of
 $100^{(\Delta m/5)} = 100^{(0.9/5)} = 2.3$

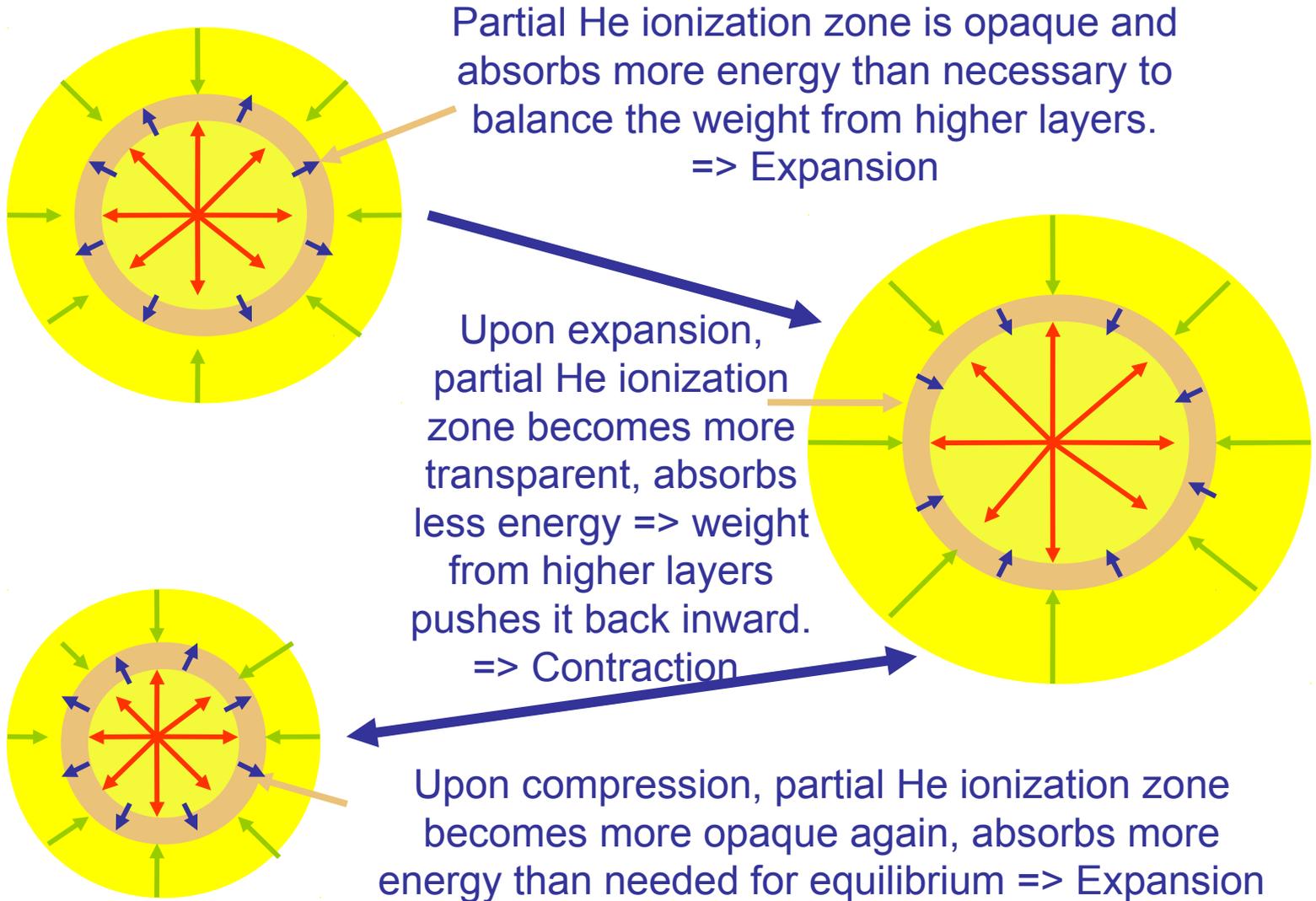
How to Find the Distance to a Pulsating Star

- Find the star's apparent magnitude m (just by looking)
- Measure the star's period (bright-dim-bright)
- Use the Period-Luminosity relation to find the stars absolute magnitude M
- Calculate the star's distance (in parsecs) using

$$d \text{ (pc)} = 10^{(m-M+5)/5}$$

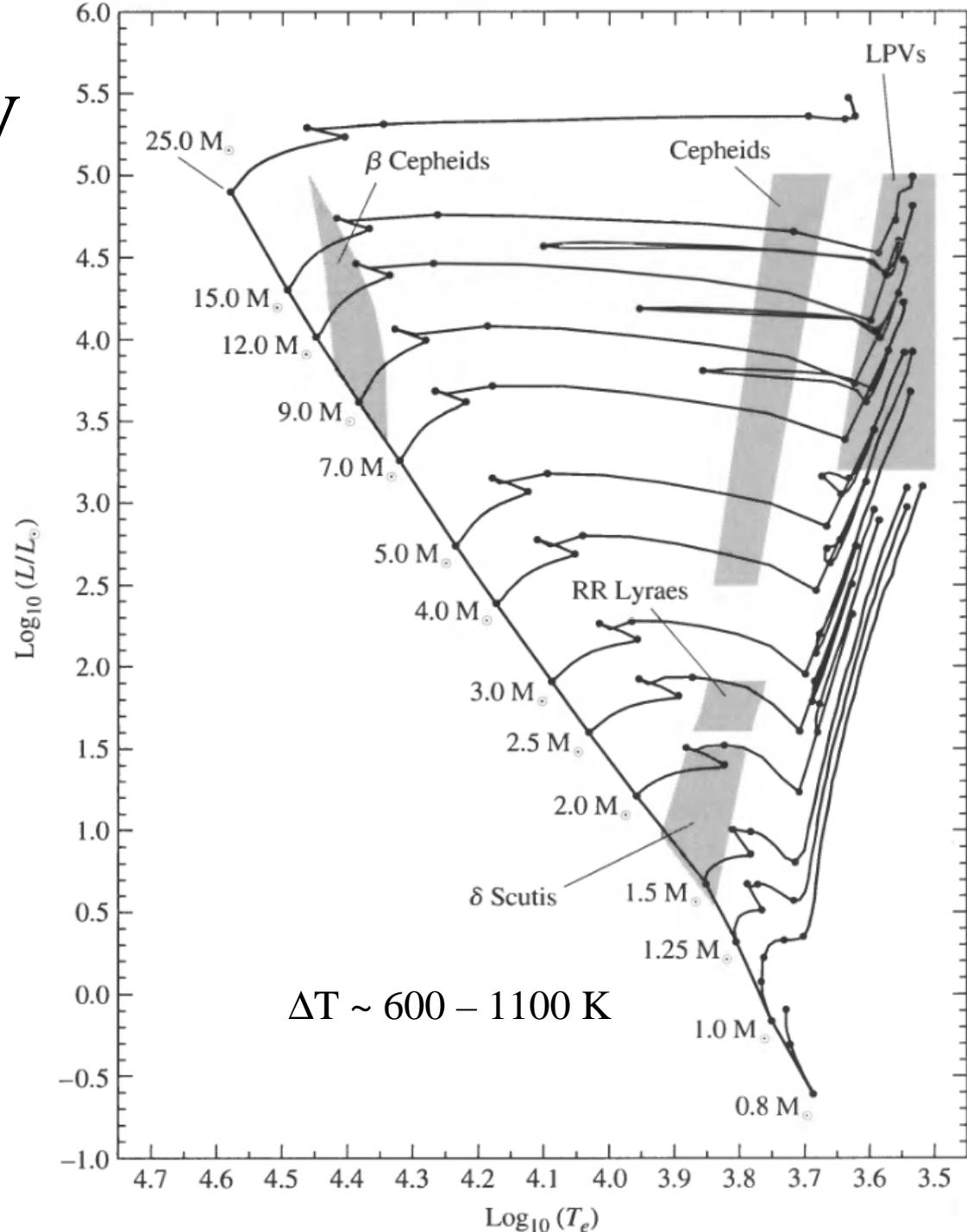


Pulsating Variables: The Valve Mechanism

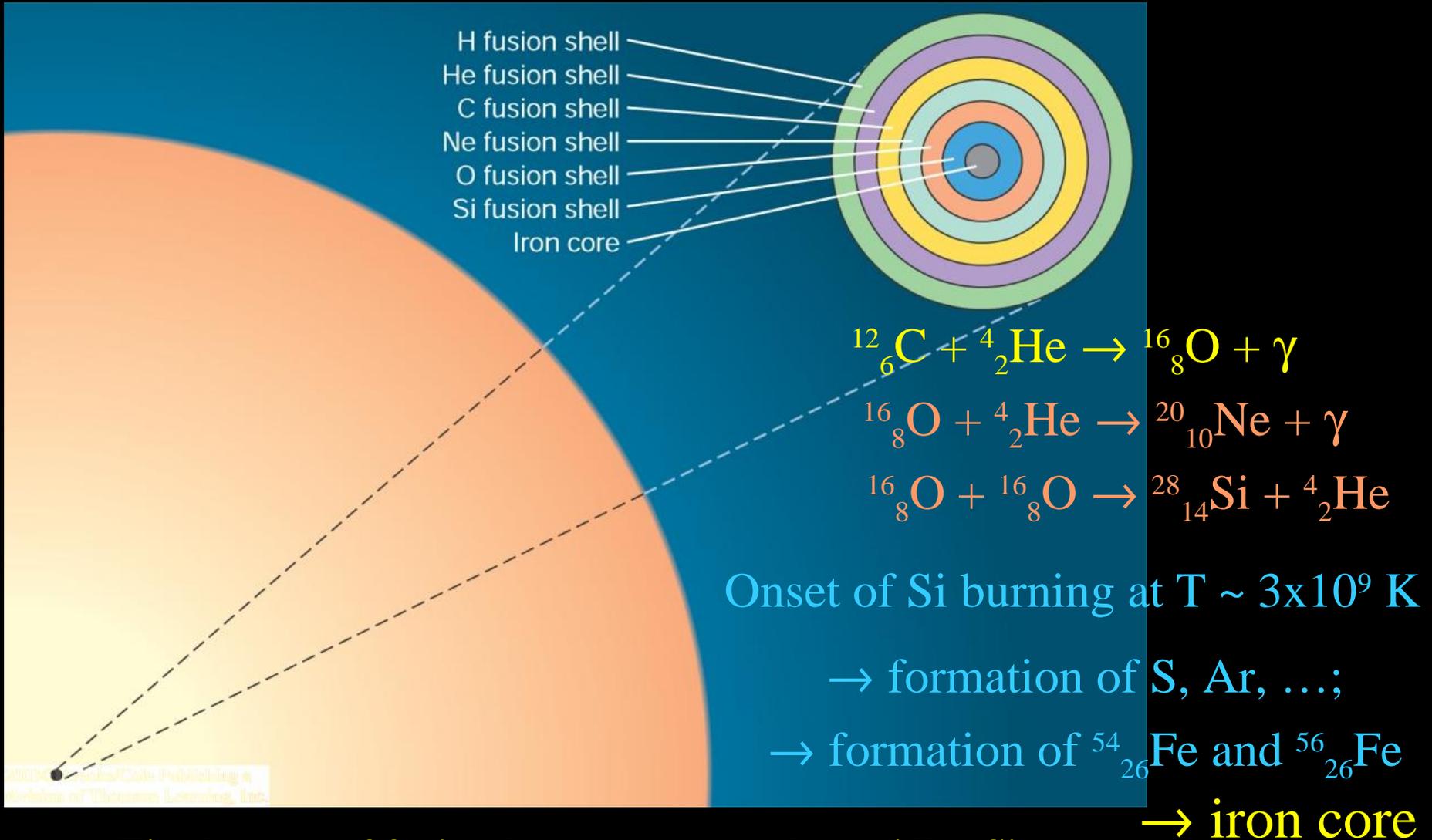


The Instability Strip

- Majority of pulsating stars lie in the instability strip on the H-R diagram
- As stars evolve along these tracks they begin to pulsate as they enter the instability strip and cease oscillations once they leave it.

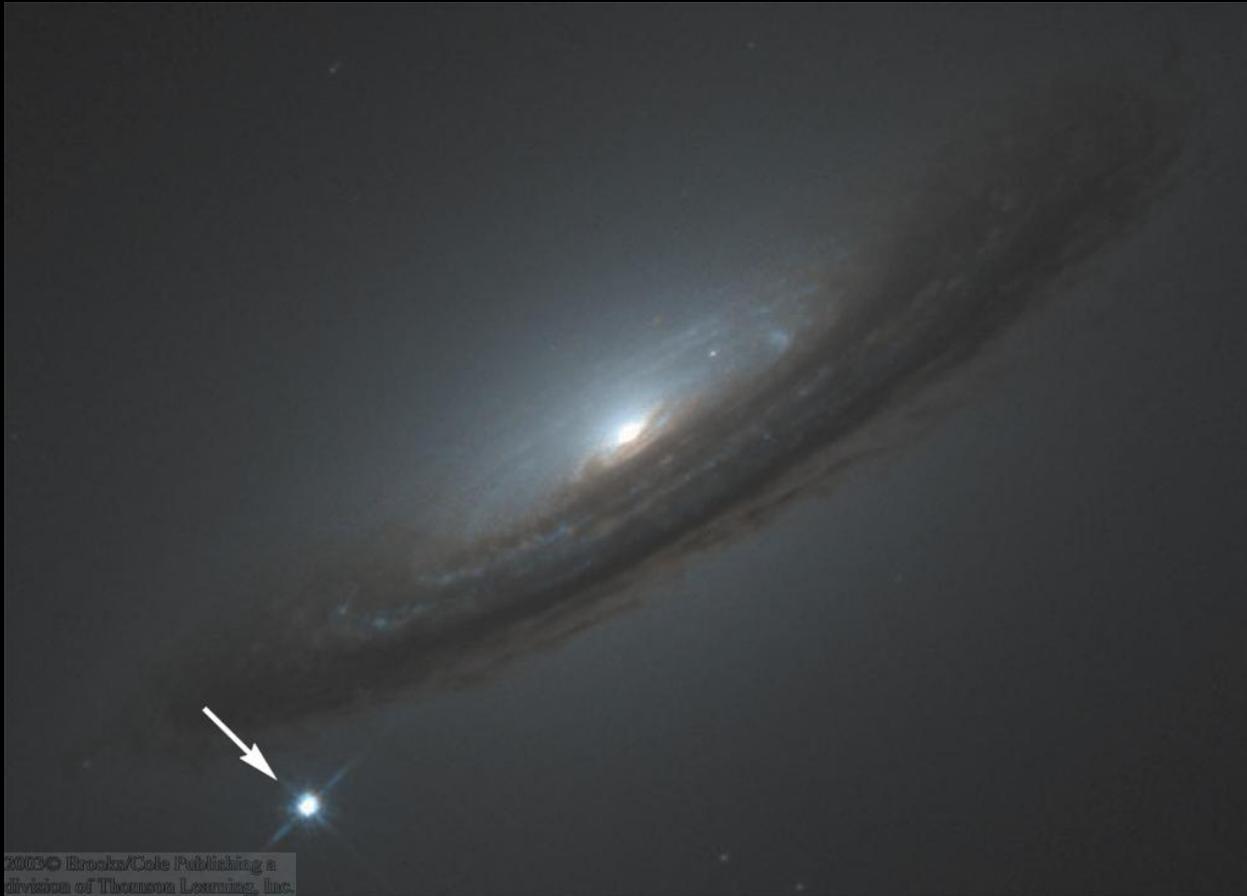


Fusion of Heavier Elements



Final stages of fusion happen extremely rapidly: Si burning lasts only for ~ 2 days.

Observations of Supernovae



Supernovae can easily be seen
in distant galaxies.

Total energy output:

$$\Delta E_{\text{ve}} \sim 3 \times 10^{46} \text{ J}$$

($\sim 100 L_0 t_{\text{life},0}$)

$$\Delta E_{\text{kin}} \sim 10^{44} \text{ J}$$

$$\Delta E_{\text{ph}} \sim 10^{42} \text{ J}$$

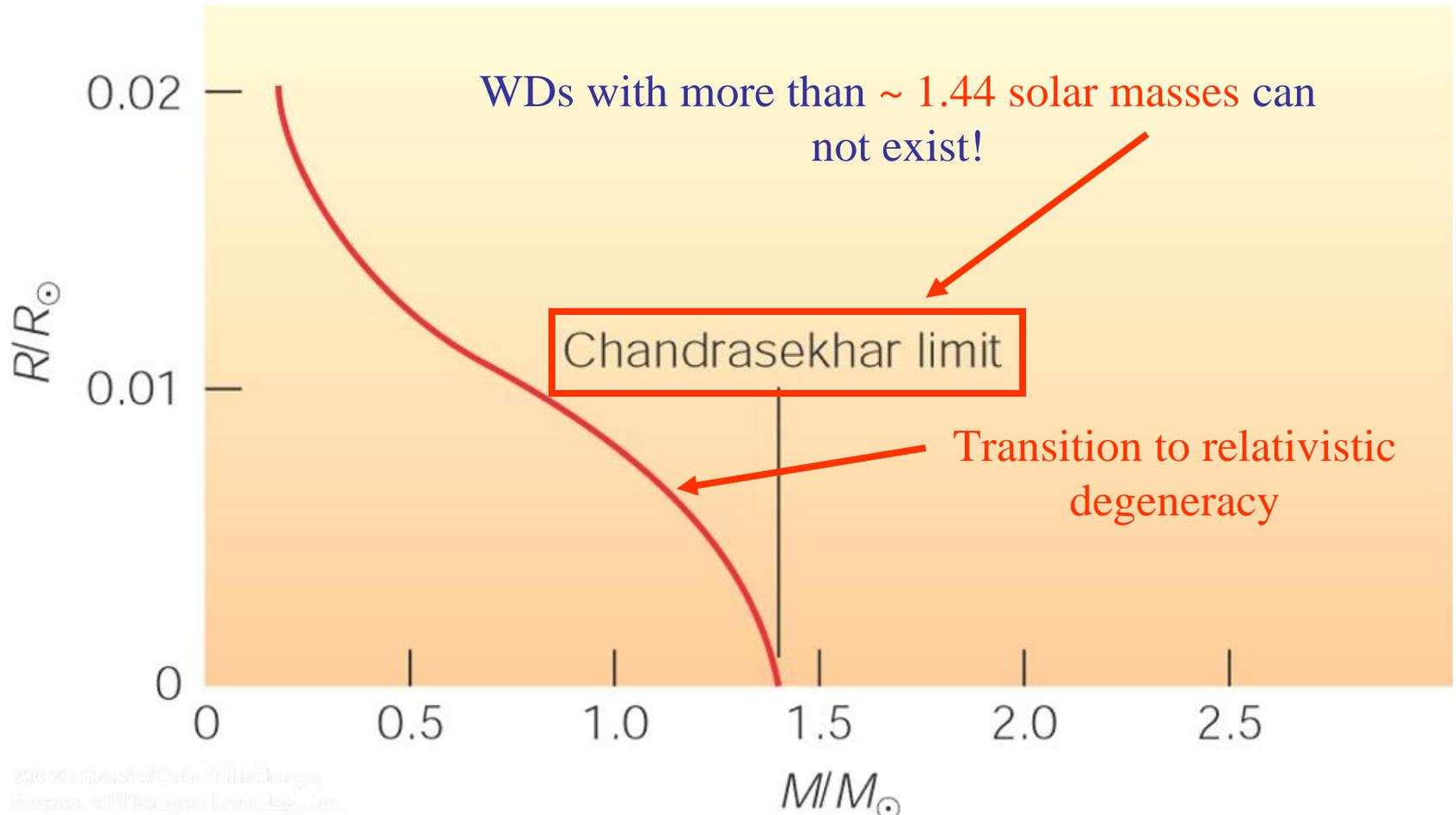
$$L_{\text{pk}} \sim 10^{36} \text{ J/s} \sim$$
$$10^9 L_0$$

$$\sim L_{\text{galaxy}}!$$

The Chandrasekhar Limit

The **more massive** a white dwarf, the **smaller** it is.

$$R_{\text{WD}} \sim M_{\text{WD}}^{-1/3} \Rightarrow M_{\text{WD}} V_{\text{WD}} = \text{const.} \quad (\text{non-rel.})$$



Properties of Neutron Stars

Typical size: $R \sim 10 \text{ km}$

Mass: $M \sim 1.4 - 3 M_{\text{sun}}$

Density: $\rho \sim 4 \times 10^{14} \text{ g/cm}^3$

→ 1 teaspoon full of NS matter has a mass
of ~ 2 billion tons!!!

Rotation periods: ~ a few ms – a few s

Magnetic fields: $B \sim 10^8 - 10^{15} \text{ G}$

(Atoll sources; ms
pulsars)

(magnetars)

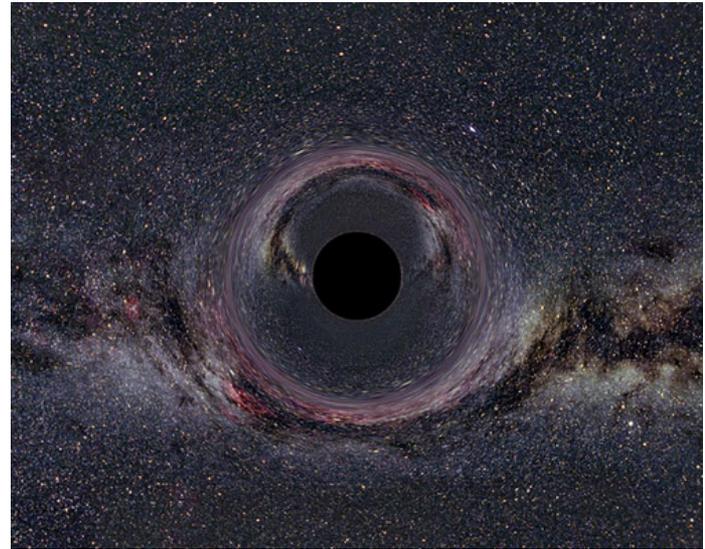
Black Holes

- In 1783 John Mitchell pondered that the escape velocity from the surface of a star 500 times larger than the sun with the same average density would equal the speed of light.



$$v_{\text{esc}} = \sqrt{2GM/r} = \sqrt{2G(500M_{\odot})/7.93R_{\odot}} = c$$

- $R=2.95(M/M_{\text{sun}})$ km!



In 1939 J. Robert Oppenheimer and Hartland Snyder described the ultimate gravitational collapse of a massive star that has exhausted its sources of nuclear fusion. They pondered what happened to the cores of stars whose mass exceeded the limit of neutron stars..

In 1967 the term “black hole” was coined
By John Archibald Wheeler